An Introduction to Convolutional Neural Networks

Yuan YAO

HKUST

Summary

- We had covered so far
  - Linear models (linear and logistic regression) – always a good start, simple yet powerful
  - Model Assessment and Selection – basics for all methods
  - Trees, Random Forests, and Boosting – good for high dim mixed-type features
  - Support Vector Machines – good for small amount of data but high dim geometric features
- Next, neural networks for unstructured data (image, language etc.):
  - **Convolutional Neural Networks** – image data
  - Generative models and GANs – new unsupervised learning for image, etc.
  - Recurrent Neural Networks, LSTM – sequence data
  - Transformer, BERT – machine translation etc.
  - Reinforcement Learning – Markov decision process, playing games, etc.

Kaggle survey: Top ML Methods

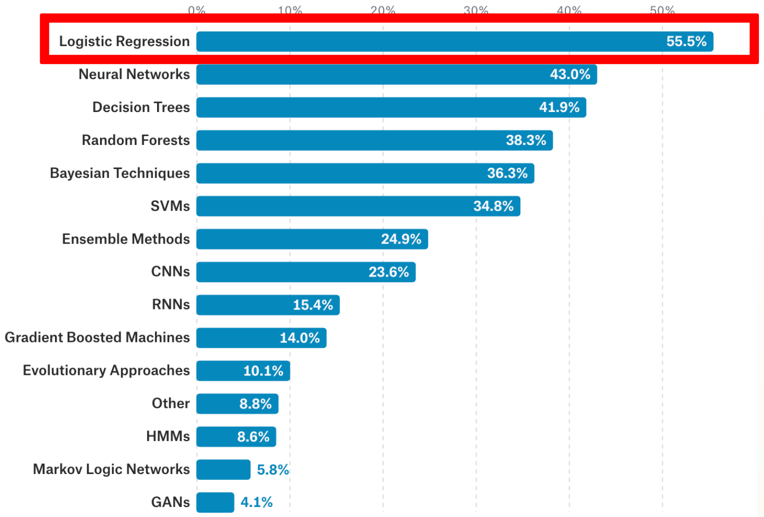
https://www.kaggle.com/surveys/2017

Academic

What data science methods are used at work?

Logistic regression is the most commonly reported data science method used at work for all industries *except* Military and Security where Neural Networks are used slightly more frequently.

Company Size | Academic | Job Title



1,201 responses

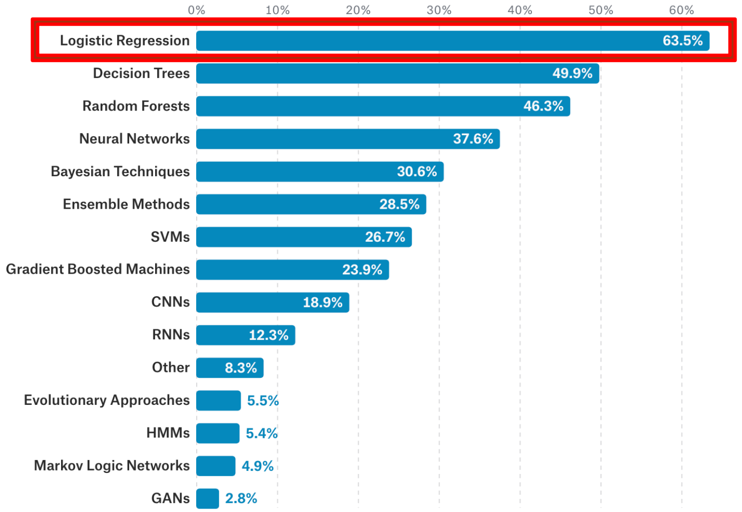
View code in Kaggle Kernels

Industry

What data science methods are used at work?

Logistic regression is the most commonly reported data science method used at work for all industries *except* Military and Security where Neural Networks are used slightly more frequently.

Company Size | Industry | Job Title



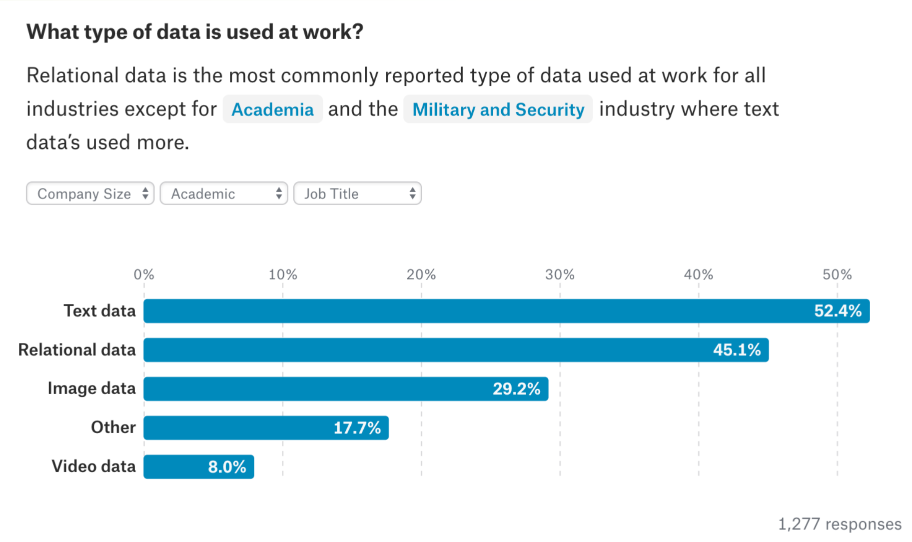
7,301 responses

View code in Kaggle Kernels

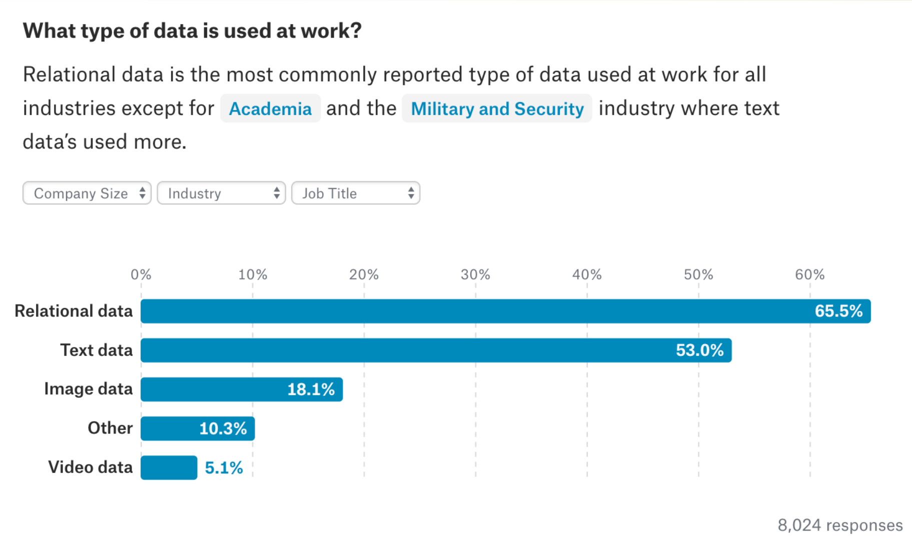
What type of data is used at work?

https://www.kaggle.com/surveys/2017

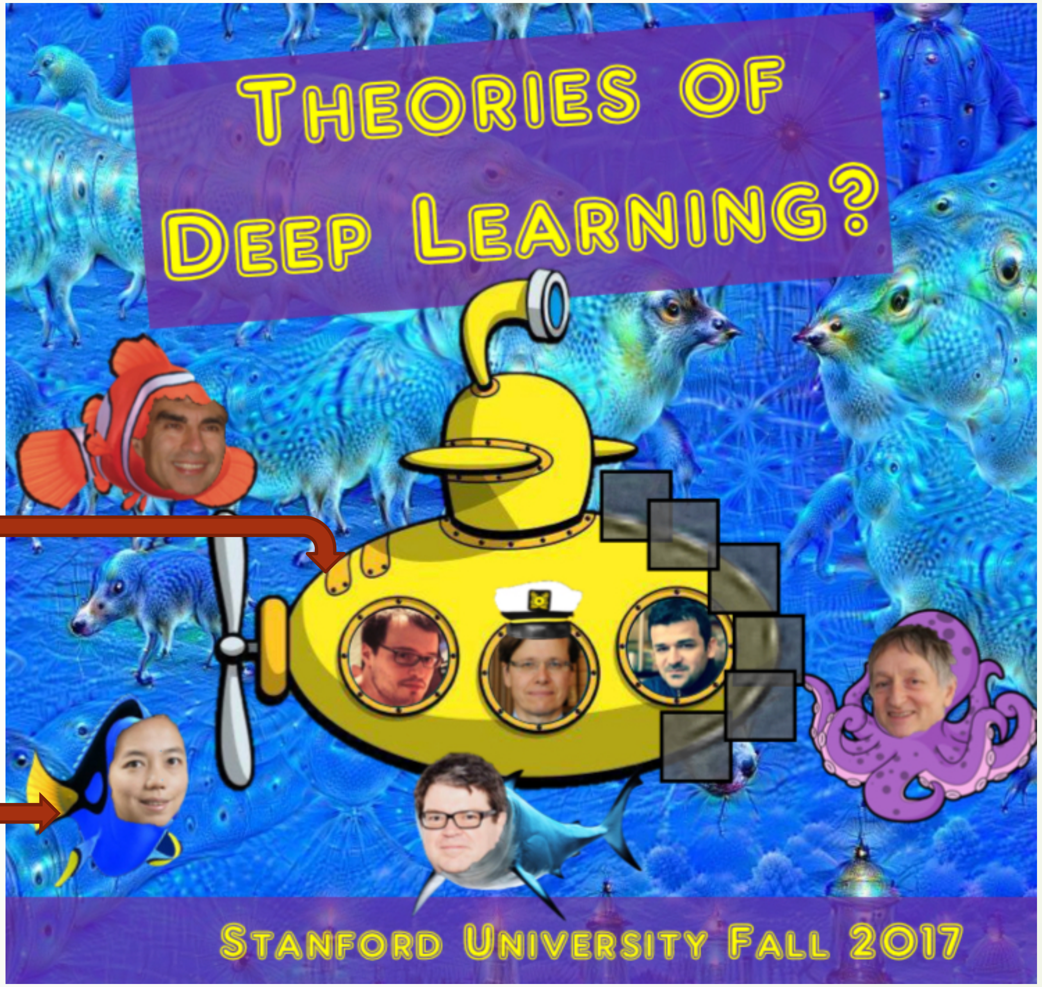
Academic



Industry



Acknowledgement



https://stats385.github.io/

http://cs231n.github.io/

A following-up course at HKUST: https://deeplearning-math.github.io/

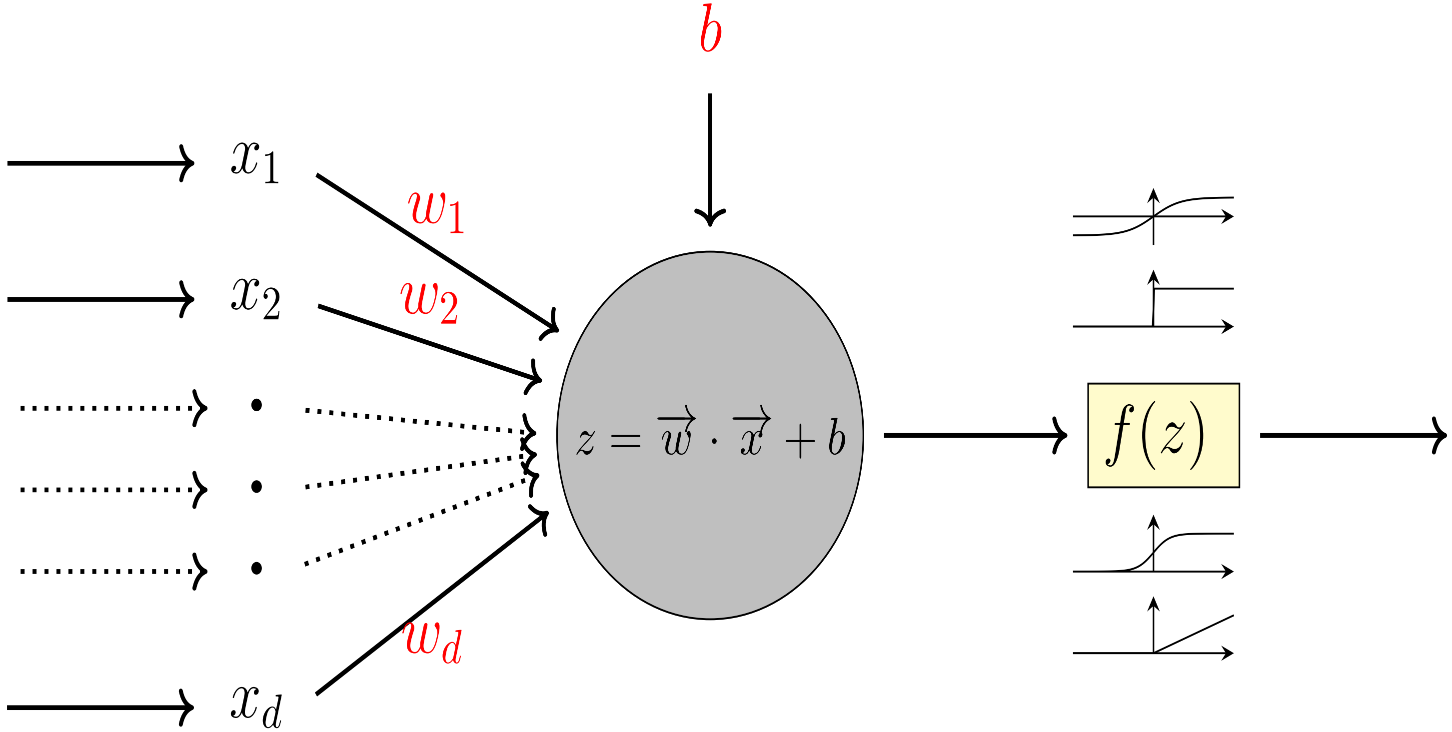
Some reference books

- Deep Learning with Python, Manning Publications 2017
  - by François Chollet
  - https://www.manning.com/books/deep-learning-with-python?a_aid=keras&a_bid=76564dff
- Deep Learning, MIT Press 2016
  - By Ian Goodfellow, Yoshua Bengio, and Aaron Courville,
  - http://www.deeplearningbook.org/
- Many other public resources

A Brief History of Neural Networks

Perceptron: single-layer

- Invented by Frank Rosenblatt (1957)



The Perceptron Algorithm

$$\ell(w) = -\sum_{i \in \mathcal{M}_w} y_i \langle w, \mathbf{x}_i \rangle, \quad \mathcal{M}_w = \{i : y_i \langle \mathbf{x}_i, w \rangle < 0, y_i \in \{-1, 1\}\}.$$

The Perceptron Algorithm is a *Stochastic Gradient Descent* method (Robbins–Monro 1950; Kiefer-Wolfowitz 1951) :

$$\begin{aligned} w_{t+1} &= w_t - \eta_t \nabla_i \ell(w) \\ &= \begin{cases} w_t - \eta_t y_i \mathbf{x}_i, & \text{if } y_i w_t^T \mathbf{x}_i < 0, \\ w_t, & \text{otherwise.} \end{cases} \end{aligned}$$

Finite Stop of Perceptron for Separable Data

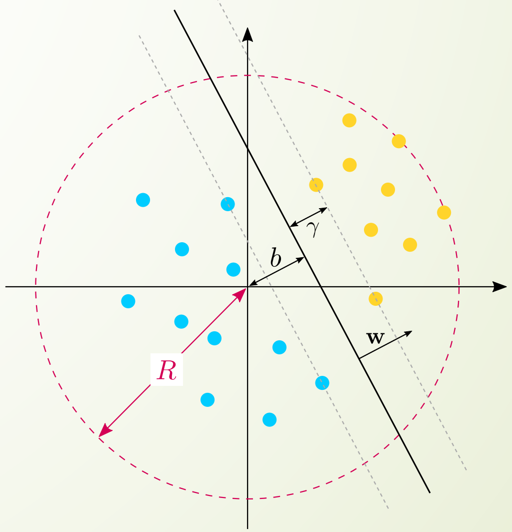
The perceptron convergence theorem was proved by Block (1962) and Novikoff (1962). The following version is based on that in Cristianini and Shawe-Taylor (2000).

Theorem 1 (Block, Novikoff). *Let the training set $S = \{(\mathbf{x}_1, t_1), \ldots, (\mathbf{x}_n, t_n)\}$ be contained in a sphere of radius R about the origin. Assume the dataset to be linearly separable, and let* $\mathbf{w}_{\text{opt}}$, $\|\mathbf{w}_{\text{opt}}\| = 1$, *define the hyperplane separating the samples, having functional margin* $\gamma > 0$. *We initialise the normal vector as* $\mathbf{w}_0 = \mathbf{0}$. *The number of updates, k, of the perceptron algorithms is then bounded by*

$$k \leq \left(\frac{2R}{\gamma}\right)^2. \tag{10}$$

Input ball: $R = \max_i \|\mathbf{x}_i\|.$

Margin: $\gamma := \min_i y_i f(x_i)$



Proof.

Proof. Though the proof can be done using the augmented normal vector and samples defined in the beginning, the notation will be a lot easier if we introduce a different augmentation: $\hat{\mathbf{w}} = (\mathbf{w}^\top, b/R)^\top = (w_1, \ldots, w_D, b/R)^\top$ and $\hat{\mathbf{x}} = (\mathbf{x}^\top, R)^\top = (x_1, \ldots, x_D, R)^\top$.

Proof (continued, growth of $|w_k|$)

We first derive an upper bound on how fast the normal vector grows. As the hyperplane is unchanged if we multiply $\hat{\mathbf{w}}$ by a constant, we can set $\eta = 1$ without loss of generality. Let $\hat{\mathbf{w}}_{k+1}$ be the updated (augmented) normal vector after the kth error has been observed.

$$\|\hat{\mathbf{w}}_{k+1}\|^2 = (\hat{\mathbf{w}}_k + t_i\hat{\mathbf{x}}_i)^{\mathsf{T}}(\hat{\mathbf{w}}_k + t_i\hat{\mathbf{x}}_i) \tag{11}$$

$$= \hat{\mathbf{w}}_k^{\mathsf{T}}\hat{\mathbf{w}}_k + \hat{\mathbf{x}}_i^{\mathsf{T}}\hat{\mathbf{x}}_i + 2t_i\hat{\mathbf{w}}_k^{\mathsf{T}}\hat{\mathbf{x}}_i \tag{12}$$

$$= \|\hat{\mathbf{w}}_k\|^2 + \|\hat{\mathbf{x}}_i\|^2 + 2t_i\hat{\mathbf{w}}_k^{\mathsf{T}}\hat{\mathbf{x}}_i\,. \tag{13}$$

Since an update was triggered, we know that $t_i\hat{\mathbf{w}}_k^{\mathsf{T}}\hat{\mathbf{x}}_i \leq 0$, thus

$$\|\hat{\mathbf{w}}_k\|^2 + \|\hat{\mathbf{x}}_i\|^2 + 2t_i\hat{\mathbf{w}}_k^{\mathsf{T}}\hat{\mathbf{x}}_i \leq \|\hat{\mathbf{w}}_k\|^2 + \|\hat{\mathbf{x}}_i\|^2 \tag{14}$$

$$= \|\hat{\mathbf{w}}_k\|^2 + (\|\mathbf{x}_i\|^2 + R^2) \tag{15}$$

$$\leq \|\hat{\mathbf{w}}_k\|^2 + 2R^2\,. \tag{16}$$

This implies that $\|\hat{\mathbf{w}}_k\|^2 \leq 2kR^2$, thus

$$\|\hat{\mathbf{w}}_{k+1}\|^2 \leq 2(k+1)R^2\,. \tag{17}$$

Proof (continued, projection on w_{opt})

We then proceed to show how the inner product between an update of the normal vector and $\hat{\mathbf{w}}_{\text{opt}}$ increase with each update:

$$\begin{aligned} \hat{\mathbf{w}}_{\text{opt}}^{\mathsf{T}} \hat{\mathbf{w}}_{k+1} &= \hat{\mathbf{w}}_{\text{opt}}^{\mathsf{T}} \hat{\mathbf{w}}_{k} + t_i \hat{\mathbf{w}}_{\text{opt}}^{\mathsf{T}} \hat{\mathbf{x}}_i && (18) \\ &\geq \hat{\mathbf{w}}_{\text{opt}}^{\mathsf{T}} \hat{\mathbf{w}}_{k} + \gamma && (19) \\ &\geq (k+1)\gamma\,, && (20) \end{aligned}$$

since $\hat{\mathbf{w}}_{\text{opt}}^{\mathsf{T}} \hat{\mathbf{w}}_{k} \geq k\gamma$. We therefore have

$$k^2\gamma^2 \leq (\hat{\mathbf{w}}_{\text{opt}}^{\mathsf{T}} \hat{\mathbf{w}}_{k})^2 \leq \|\hat{\mathbf{w}}_{\text{opt}}\|^2 \|\hat{\mathbf{w}}_{k}\|^2 \leq 2kR^2 \|\hat{\mathbf{w}}_{\text{opt}}\|^2\,, \tag{21}$$

where we have made use of the Cauchy-Schwarz inequality. As $k^2\gamma^2$ grows faster than $2kR^2$, Eq. (21) can hold if and only if

$$k \leq 2\|\hat{\mathbf{w}}_{\text{opt}}\|^2 \frac{R^2}{\gamma^2}\,. \tag{22}$$

Proof (continued, combined bounds)

As $b \leq R$, we can rewrite the norm of the normal vector:

$$\|\hat{\mathbf{w}}_{\text{opt}}\|^2 = \|\mathbf{w}_{\text{opt}}\|^2 + \frac{b^2}{R^2} \leq \|\mathbf{w}_{\text{opt}}\|^2 + 1 = 2. \tag{23}$$

The bound on k now becomes

$$k \leq 4\frac{R^2}{\gamma^2} = \left(\frac{2R}{\gamma}\right)^2, \tag{24}$$

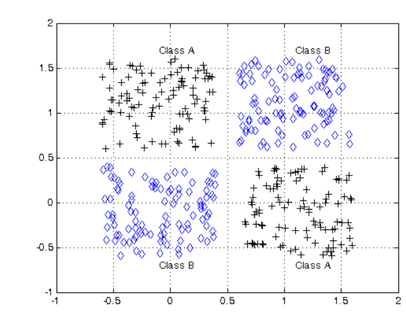
which therefore bounds the number of updates necessary to find the separating hyperplane. □

Locality or Sparsity of Computation

Minsky and Papert, 1969

Perceptron can't do **XOR** classification

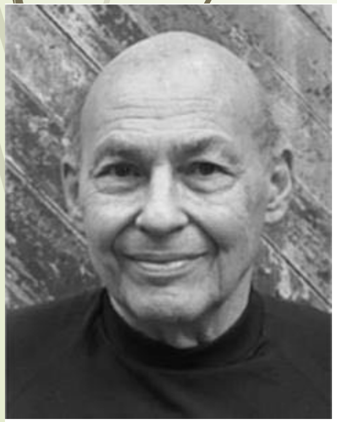
Perceptron needs infinite global information to compute **connectivity**



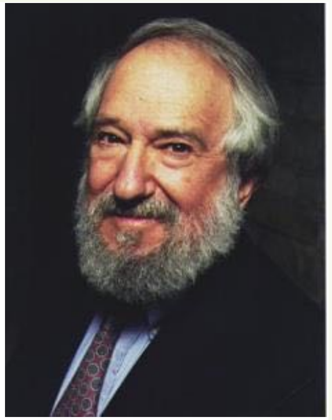
Locality or **Sparsity** is important:

Locality in time?

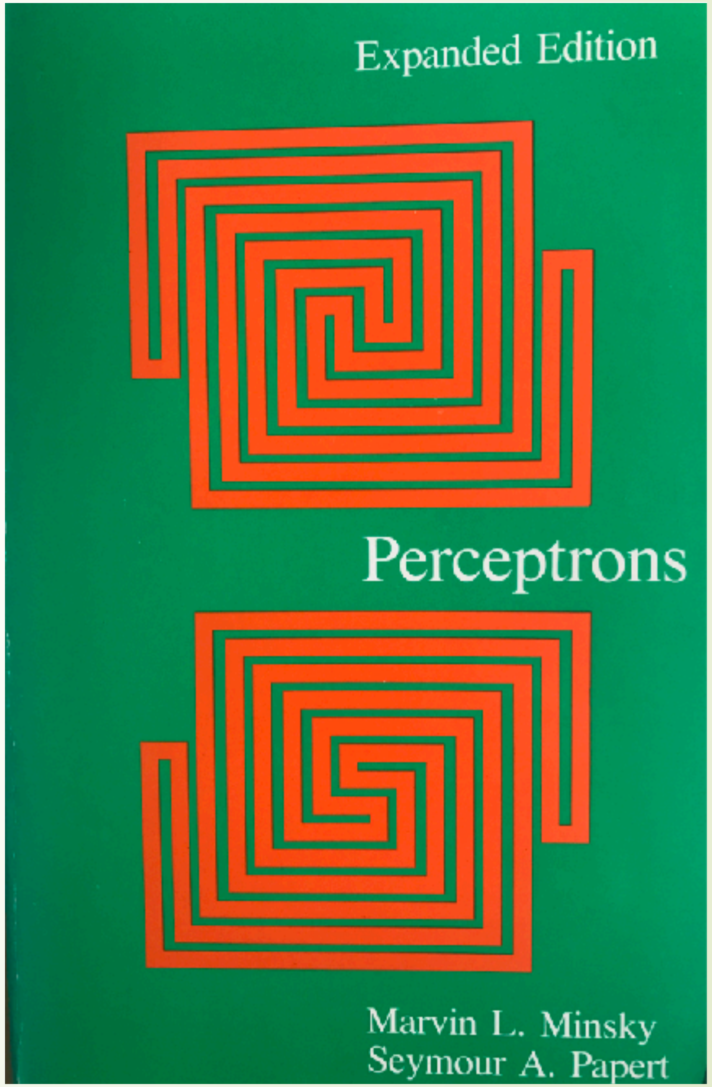
Locality in space?



Marvin Minsky



Seymour Papert



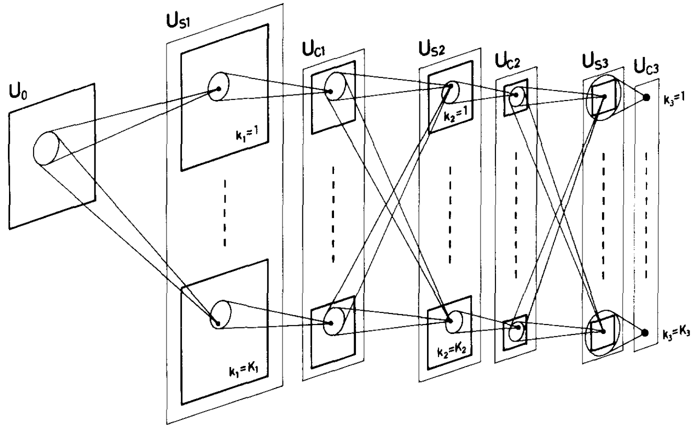
Convolutional Neural Networks: shift invariances and locality

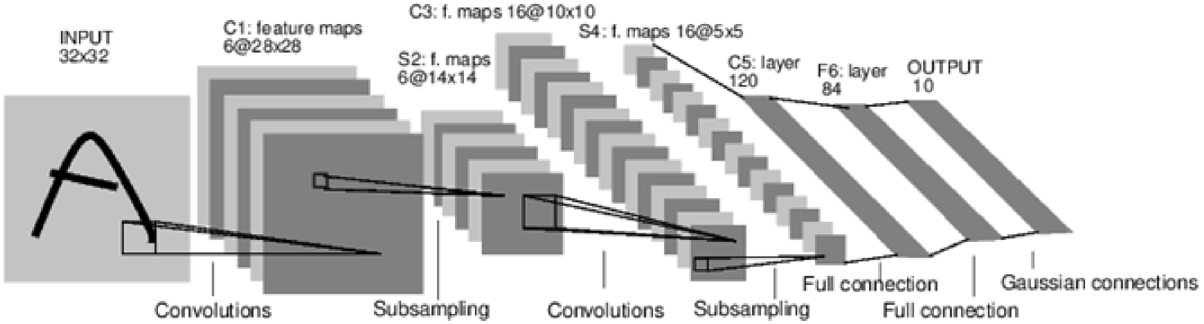
- Can be traced to *Neocognitron* of Kunihiko Fukushima (1979)
- Yann LeCun combined convolutional neural networks with back propagation (1989)
- Imposes **shift invariance** and **locality** on the weights
- Forward pass remains similar
- Backpropagation slightly changes – need to sum over the gradients from all spatial positions

Biol. Cybernetics 36, 193–202 (1980)

Neocognitron: A Self-organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected by Shift in Position

Kunihiko Fukushima
NHK Broadcasting Science Research Laboratories, Kinuta, Setagaya, Tokyo, Japan





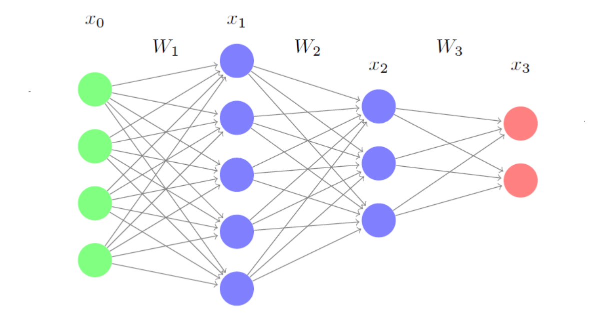
Multilayer Perceptrons (MLP) and Back-Propagation (BP) Algorithms

Rumelhart, Hinton, Williams (1986)
Learning representations by back-propagating errors, Nature, 323(9): 533-536

BP algorithms as **stochastic gradient descent** algorithms (**Robbins–Monro 1950; Kiefer-Wolfowitz 1951**) with Chain rules of Gradient maps

MLP classifies **XOR**, but the global hurdle on topology (connectivity) computation still exists

NATURE VOL. 323 9 OCTOBER 1986 LETTERS TO NATURE 533



Learning representations by back-propagating errors

David E. Rumelhart*, Geoffrey E. Hinton† & Ronald J. Williams*

* Institute for Cognitive Science, C-015, University of California, San Diego, La Jolla, California 92093, USA
† Department of Computer Science, Carnegie-Mellon University, Pittsburgh, Philadelphia 15213, USA

We describe a new learning procedure, back-propagation, for networks of neurone-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure[1].

There have been many attempts to design self-organizing neural networks. The aim is to find a powerful synaptic modification rule that will allow an arbitrarily connected neural network to develop an internal structure that is appropriate for a particular task domain. The task is specified by giving the desired state vector of the output units for each state vector of the input units. If the input units are directly connected to the output units it is relatively easy to find learning rules that iteratively adjust the relative strengths of the connections so as to progressively reduce the difference between the actual and desired output vectors[2]. Learning becomes more interesting but more difficult when we introduce hidden units whose actual or desired states are not specified by the task. (In perceptrons, there are 'feature analysers' between the input and output that are not true hidden units because their input connections are fixed by hand, so their states are completely determined by the input vector: they do not learn representations.) The learning procedure must decide under what circumstances the hidden units should be active in order to help achieve the desired input–output behaviour. This amounts to deciding what these units should represent. We demonstrate that a general purpose and relatively simple procedure is powerful enough to construct appropriate internal representations.

The simplest form of the learning procedure is for layered networks which have a layer of input units at the bottom; any number of intermediate layers; and a layer of output units at the top. Connections within a layer or from higher to lower layers are forbidden, but connections can skip intermediate layers. An input vector is presented to the network by setting the states of the input units. Then the states of the units in each layer are determined by applying equations (1) and (2) to the connections coming from lower layers. All units within a layer have their states set in parallel, but different layers have their states set sequentially, starting at the bottom and working upwards until the states of the output units are determined.

The total input, x_j, to unit j is a linear function of the outputs, y_i, of the units that are connected to j and of the weights, w_{ji}, on these connections

$$x_j = \sum_i y_i w_{ji} \quad (1)$$

Units can be given biases by introducing an extra input to each unit which always has a value of 1. The weight on this extra input is called the bias and is equivalent to a threshold of the opposite sign. It can be treated just like the other weights.

A unit has a real-valued output, y_j, which is a non-linear function of its total input

$$y_j = \frac{1}{1+e^{-x_j}} \quad (2)$$

† To whom correspondence should be addressed.

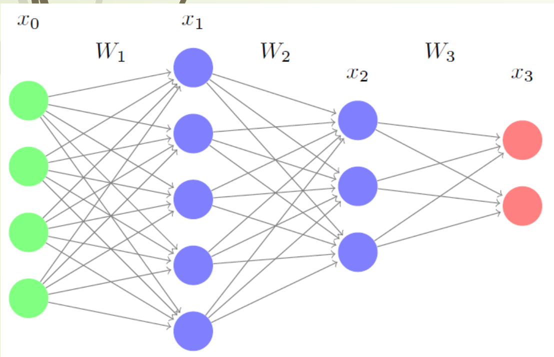
BP Algorithm: Forward Pass

- Cascade of repeated [linear operation followed by coordinatewise nonlinearity]'s
- Nonlinearities: sigmoid, hyperbolic tangent, (recently) ReLU.

Algorithm 1 Forward pass

Input: x_0
Output: x_L

1: **for** $\ell = 1$ to L **do**
2: $\quad x_\ell = f_\ell(W_\ell x_{\ell-1} + b_\ell)$
3: **end for**



BP algorithm = Gradient Descent Method

- Training examples $\{x_0^i\}_{i=1}^n$ and labels $\{y^i\}_{i=1}^n$
- Output of the network $\{x_L^i\}_{i=1}^m$
- Objective

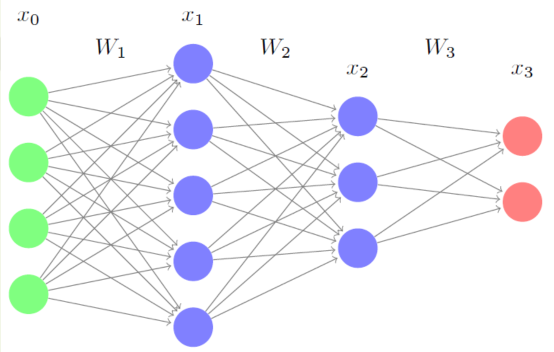
$$J(\{W_l\}, \{b_l\}) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\|y^i - x_L^i\|_2^2 \qquad (1)$$

Other losses include cross-entropy, logistic loss, exponential loss, etc.

- Gradient descent

$$W_l = W_l - \eta\frac{\partial J}{\partial W_l}$$

$$b_l = b_l - \eta\frac{\partial J}{\partial b_l}$$



In practice: use Stochastic Gradient Descent (SGD)

Derivation of BP: Lagrangian Multiplier

LeCun et al. 1988

Given n training examples $(I_i, y_i) \equiv$ (input,target) and L layers

- Constrained optimization

$$\min_{W,x} \quad \sum_{i=1}^n \|x_i(L) - y_i\|_2$$

subject to

$$x_i(\ell) = f_\ell\Big[W_\ell x_i(\ell - 1)\Big],$$

$$i = 1, \ldots, n, \quad \ell = 1, \ldots, L, \; x_i(0) = I_i$$

- Lagrangian formulation (Unconstrained)

$$\min_{W,x,B} \mathcal{L}(W, x, B)$$

$$\mathcal{L}(W, x, B) = \sum_{i=1}^n \Big\{ \|x_i(L) - y_i\|_2^2 + \sum_{\ell=1}^L B_i(\ell)^T \Big(x_i(\ell) - f_\ell\Big[W_\ell x_i(\ell - 1)\Big]\Big)\Big\}$$

http://yann.lecun.com/exdb/publis/pdf/lecun-88.pdf

back-propagation – derivation

- $\frac{\partial \mathcal{L}}{\partial B}$

Forward pass

$$x_i(\ell) = f_\ell\Big[\underbrace{W_\ell x_i\,(\ell - 1)}_{A_i(\ell)}\Big] \quad \ell = 1, \ldots, L, \quad i = 1, \ldots, n$$

- $\frac{\partial \mathcal{L}}{\partial x}$, $z_\ell = [\nabla f_\ell] B(\ell)$

Backward (adjoint) pass

$$z(L) = 2\nabla f_L\Big[A_i(L)\Big]\,(y_i - x_i(L))$$

$$z_i(\ell) = \nabla f_\ell\Big[A_i(\ell)\Big] W_{\ell+1}^T z_i(\ell + 1) \quad \ell = 0, \ldots, L - 1$$

- $W \leftarrow W + \lambda \frac{\partial \mathcal{L}}{\partial W}$

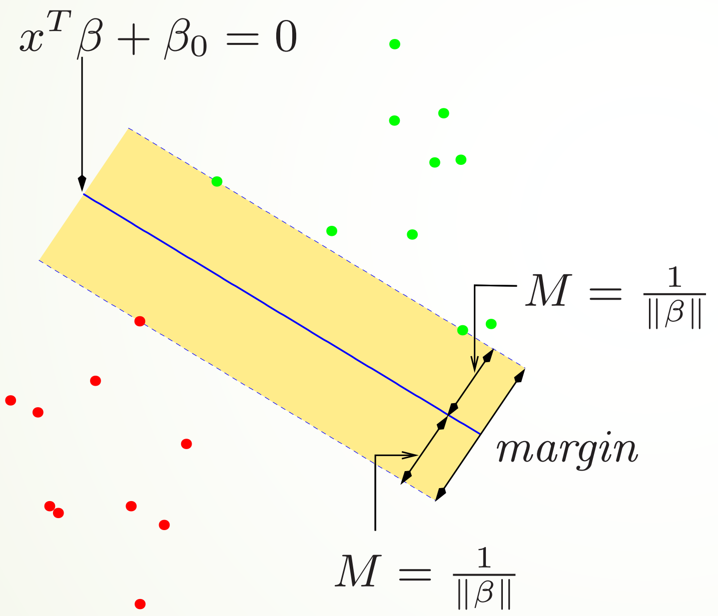
Weight update

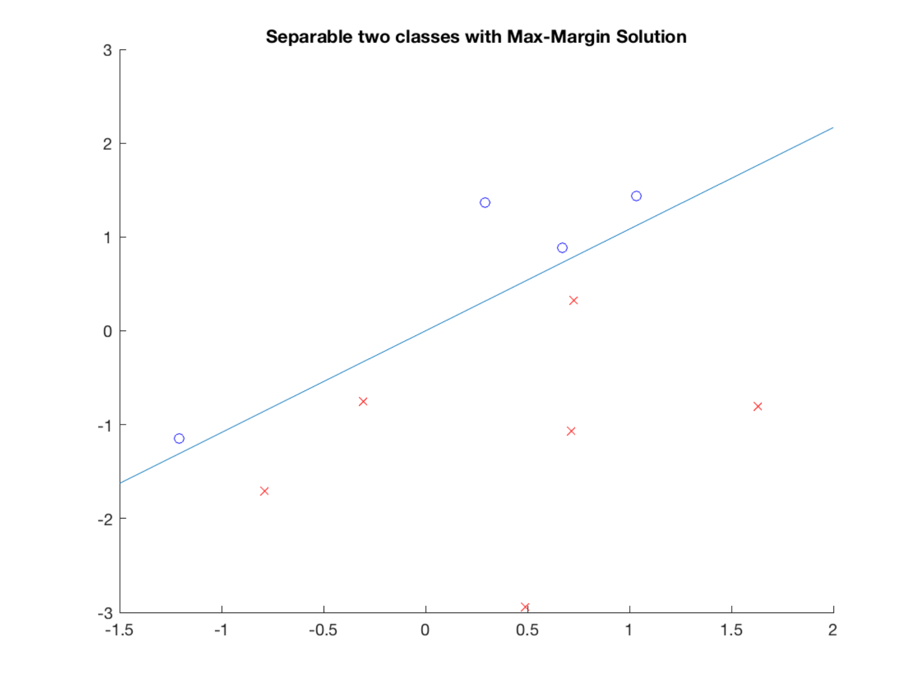
$$W_\ell \leftarrow W_\ell + \lambda \textstyle\sum_{i=1}^n z_i(\ell) x_i^T(\ell - 1)$$

Support Vector Machine (Max-Margin Classifier)

$$\text{minimize}_{\beta_0,\beta_1,\ldots,\beta_p} \|\beta\|^2 := \sum_j \beta_j^2$$

$$\text{subject to } y_i(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip}) \geq 1 \text{ for all } i$$

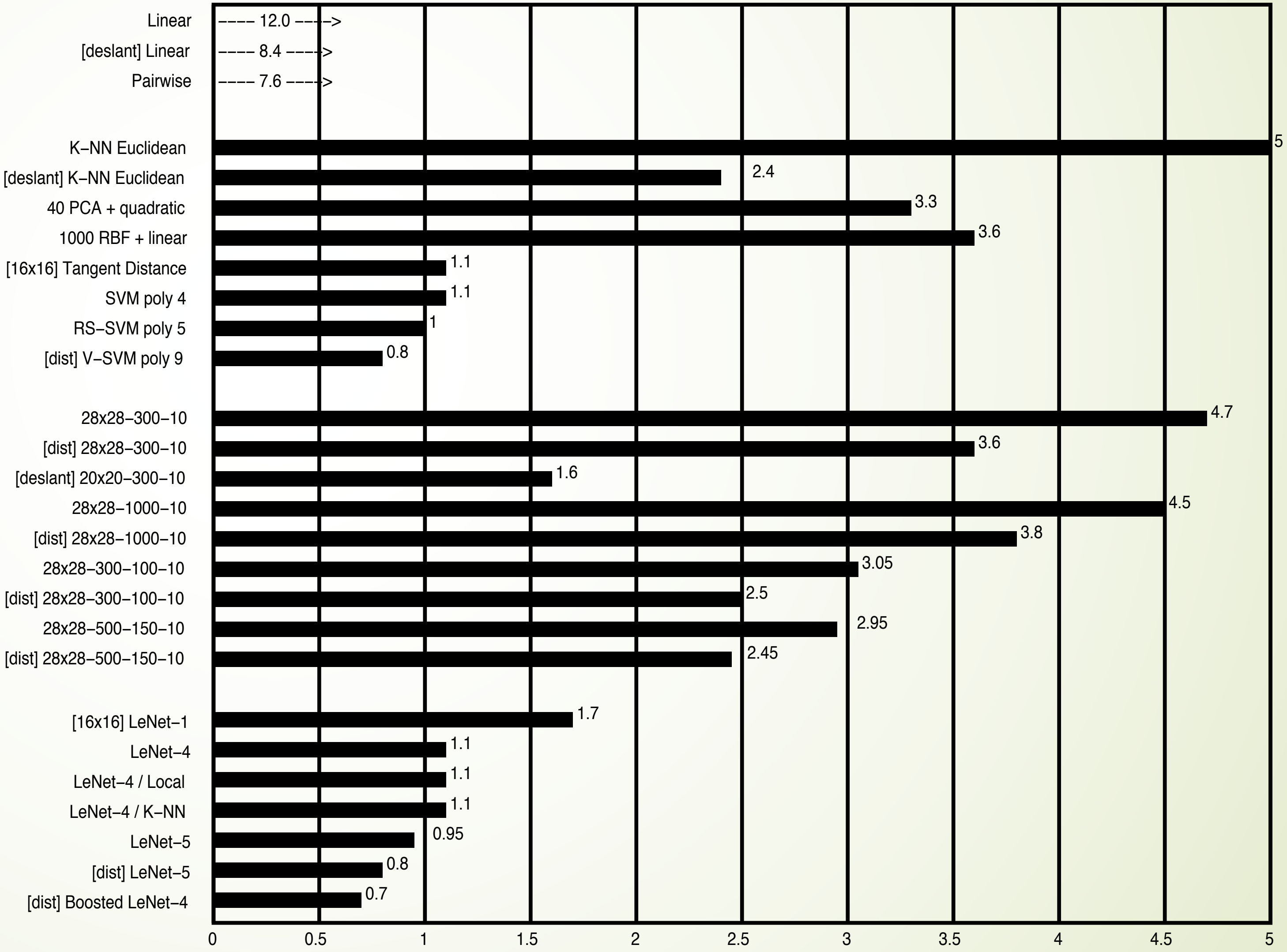




Vladmir Vapnik, 1994

Convex optimization + Reproducing Kernel Hilbert Spaces (Grace Wahba etc.)

MNIST Challenge Test Error: SVM vs. CNN LeCun et al. 1998



Simple SVM performs as well as Multilayer Convolutional Neural Networks which need careful tuning (LeNets)

Second dark era for NN: 2000s

LeNet

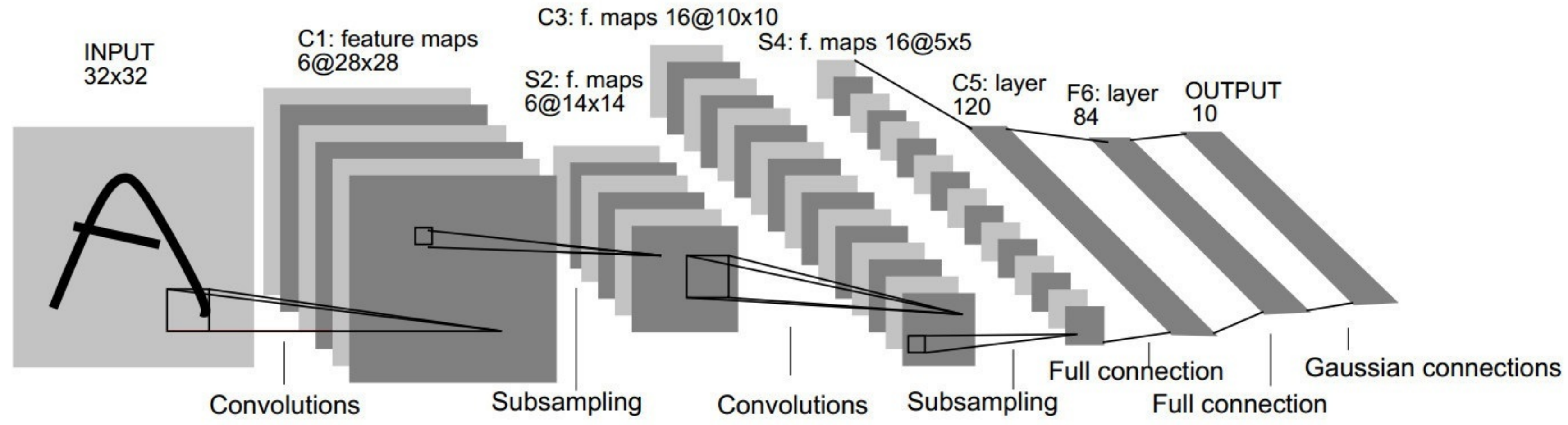
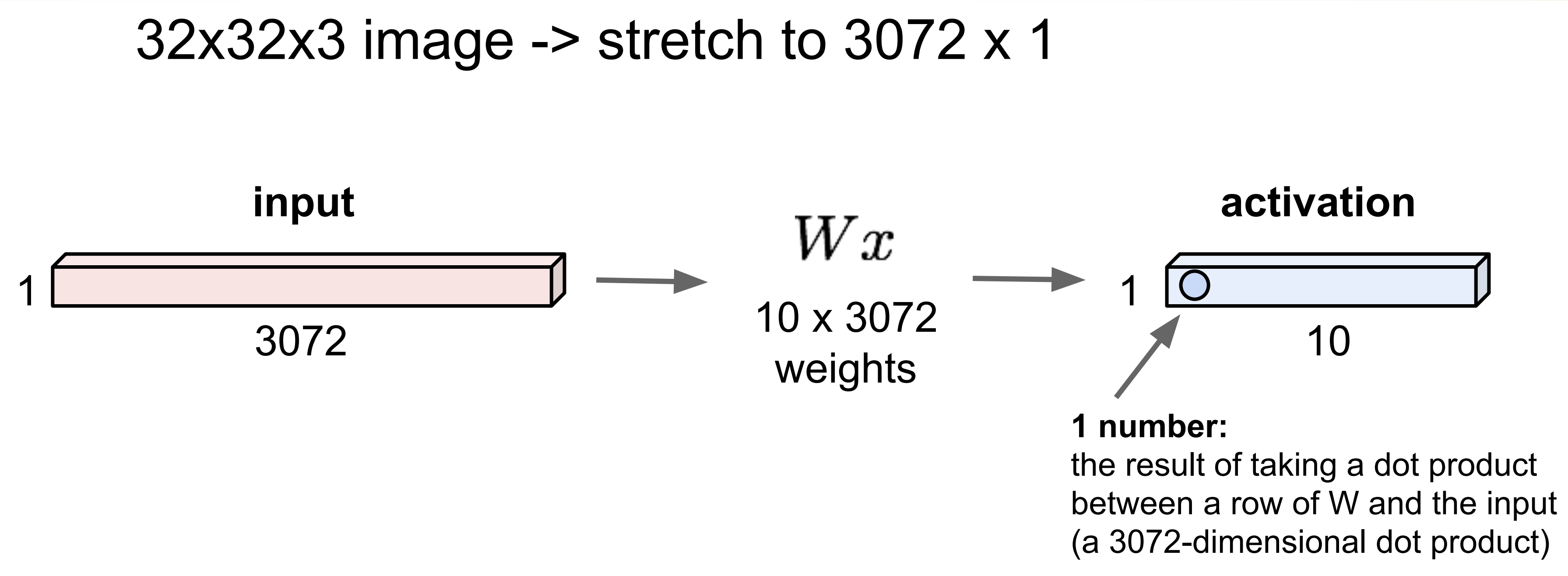


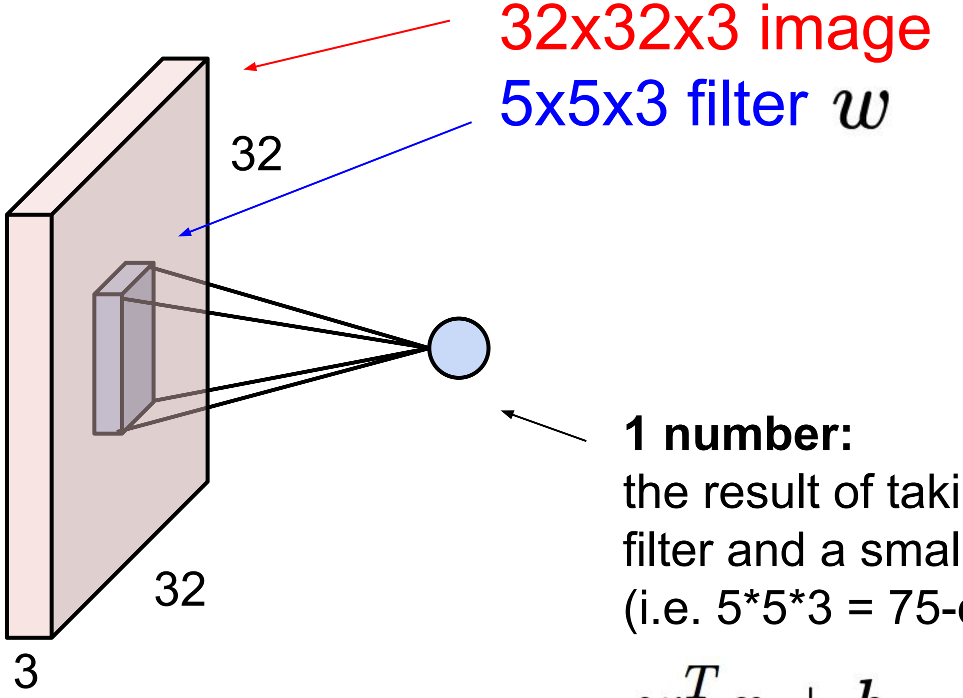
Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

- **Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner**. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, november 1998.

Fully Connected Layer



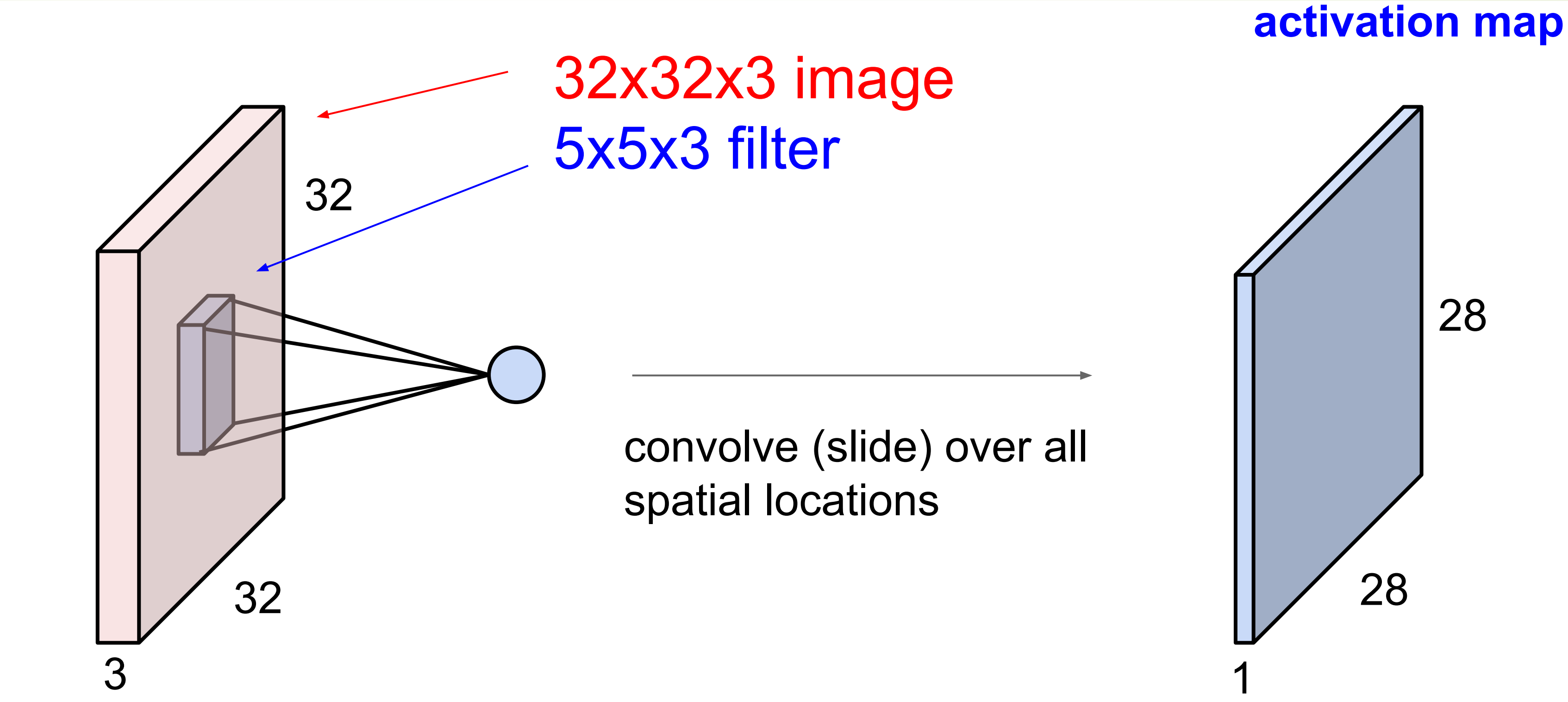
Convolution



1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

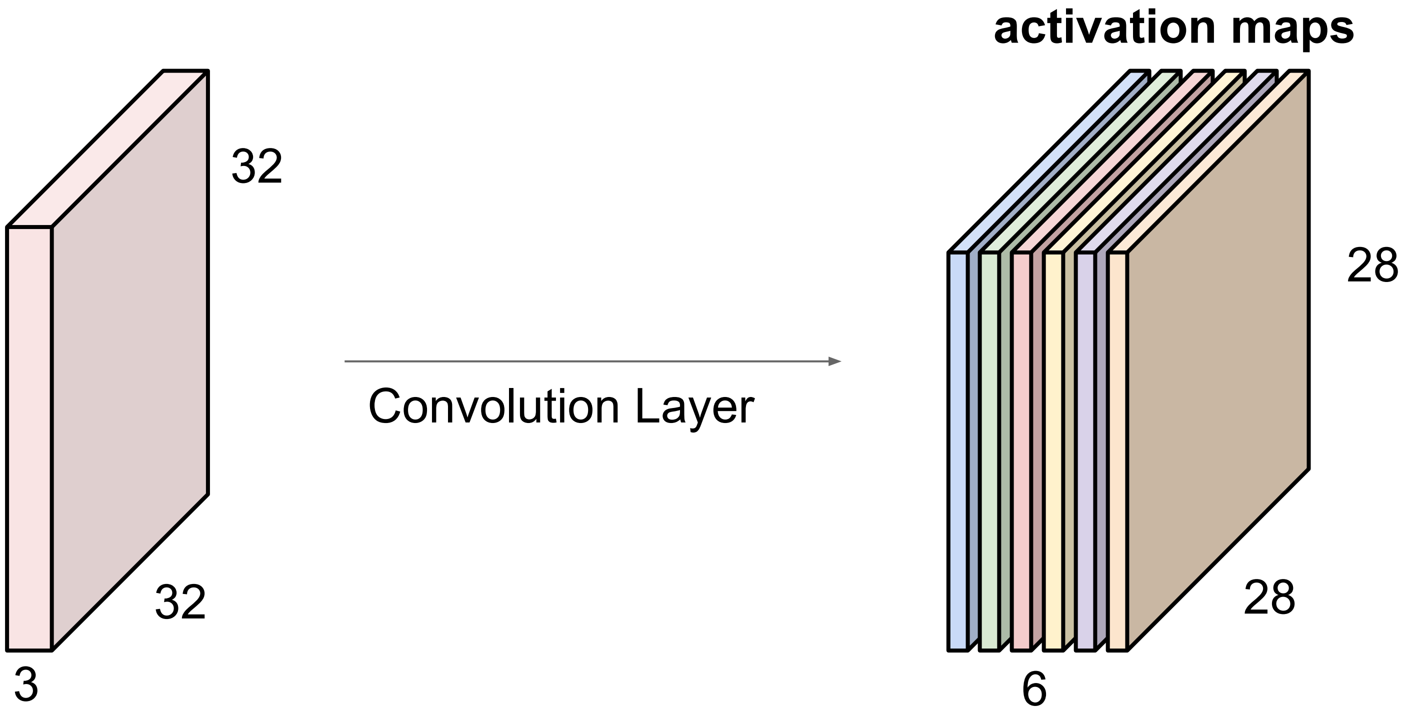
Convolution Layer: a first (blue) filter



Convolution Layer: a second (green) filter

Convolution Layer

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

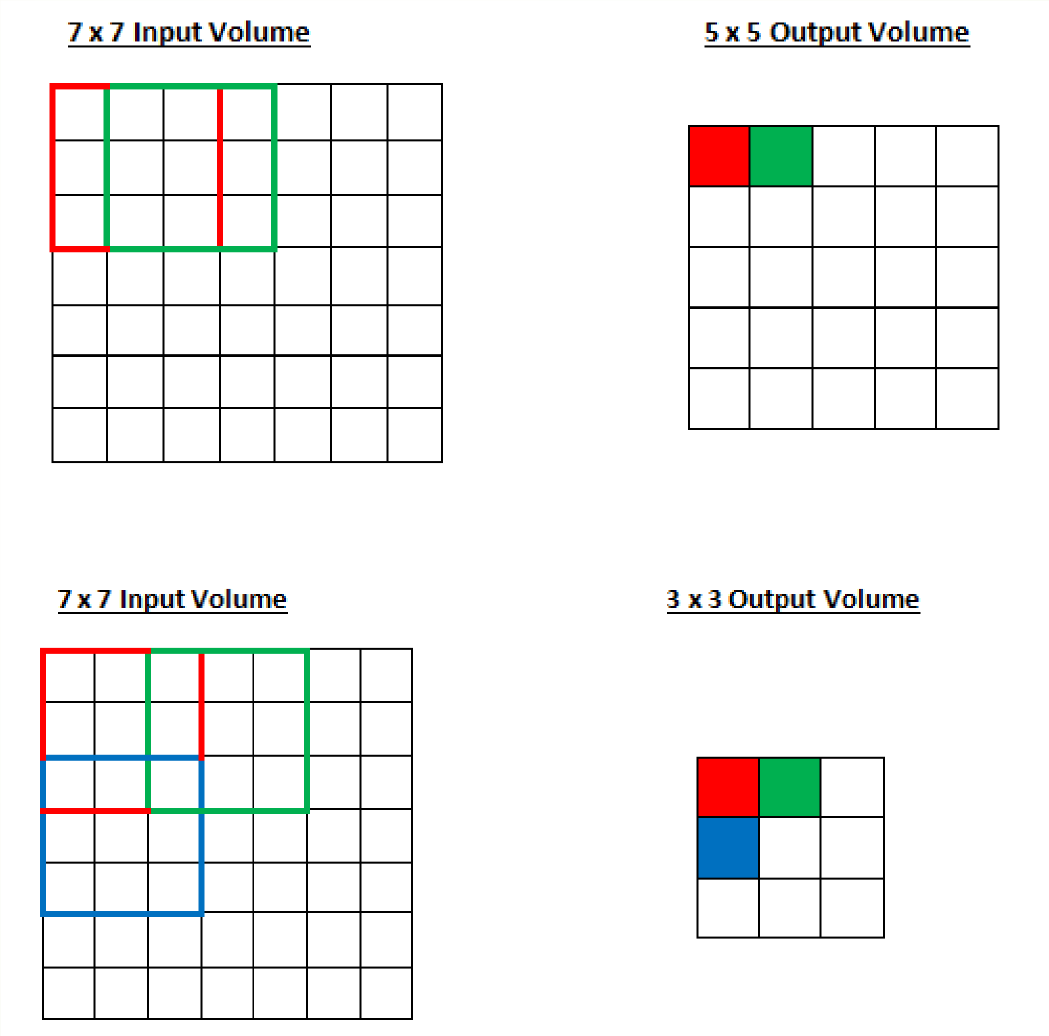


We stack these up to get a "new image" of size 28x28x6!

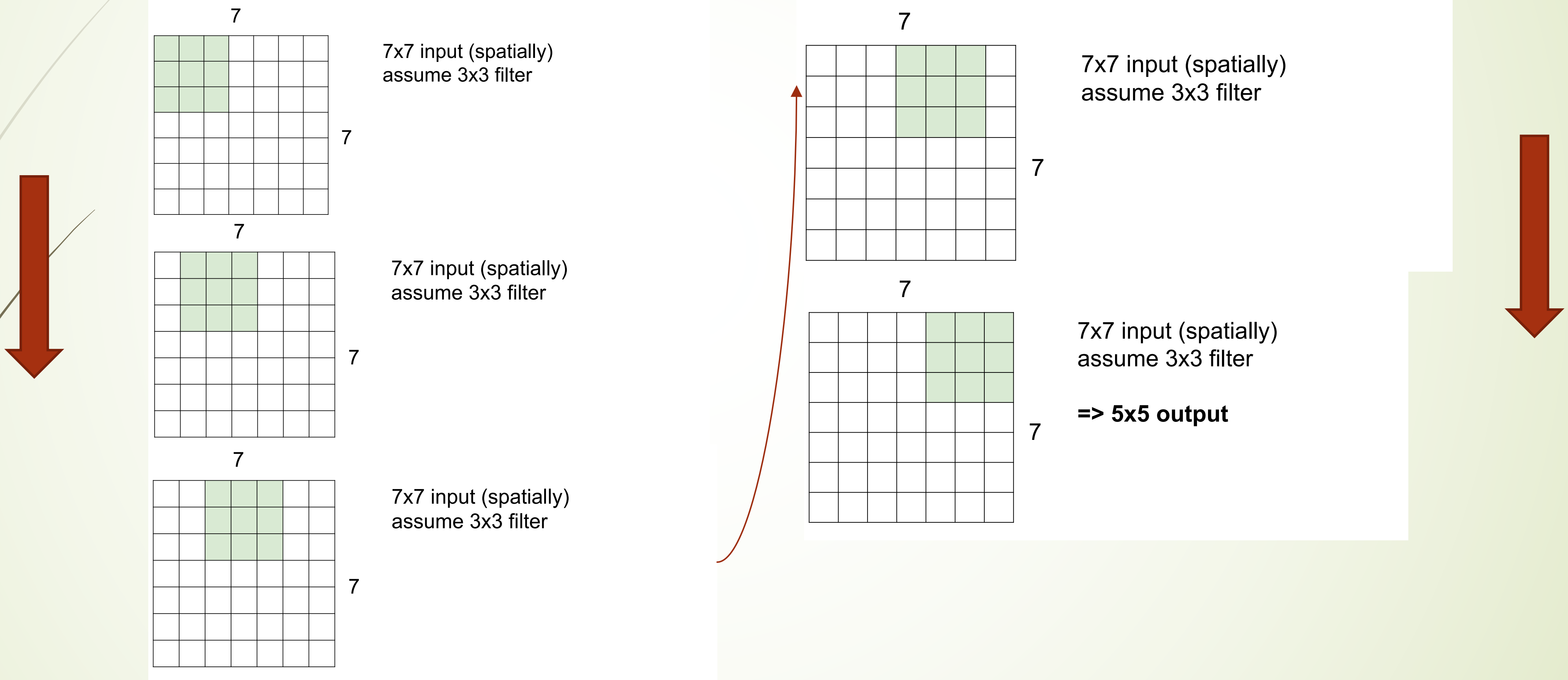
Stride

Stride 1

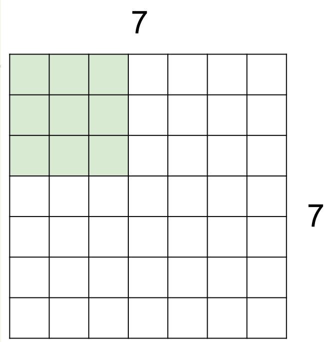
Stride 2



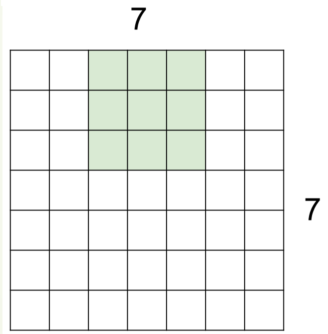
A Closer Look at Convolution: stride=1



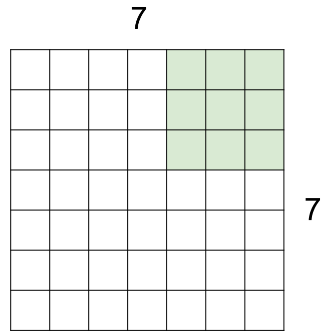
A Closer Look at Convolution: stride=2



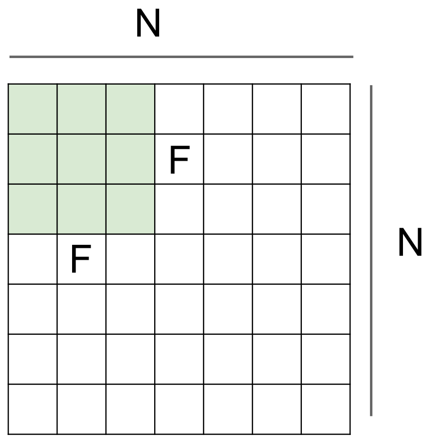
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**



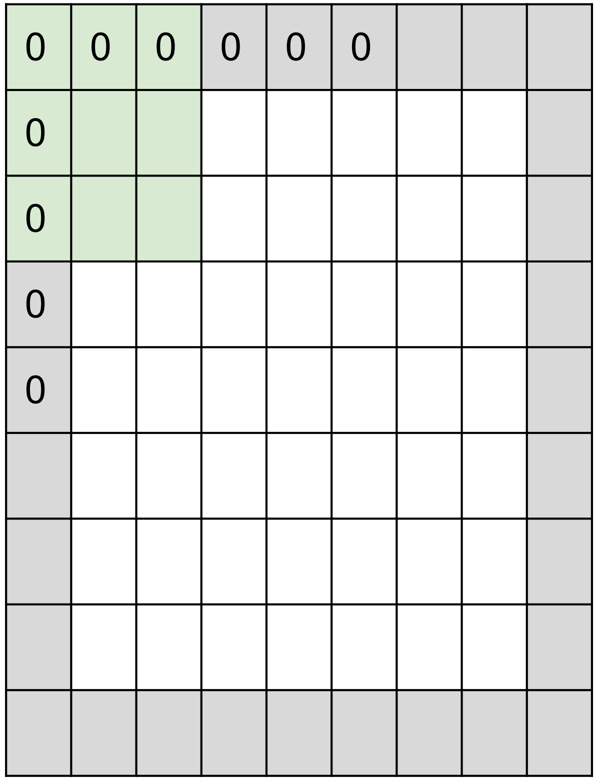
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!



Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

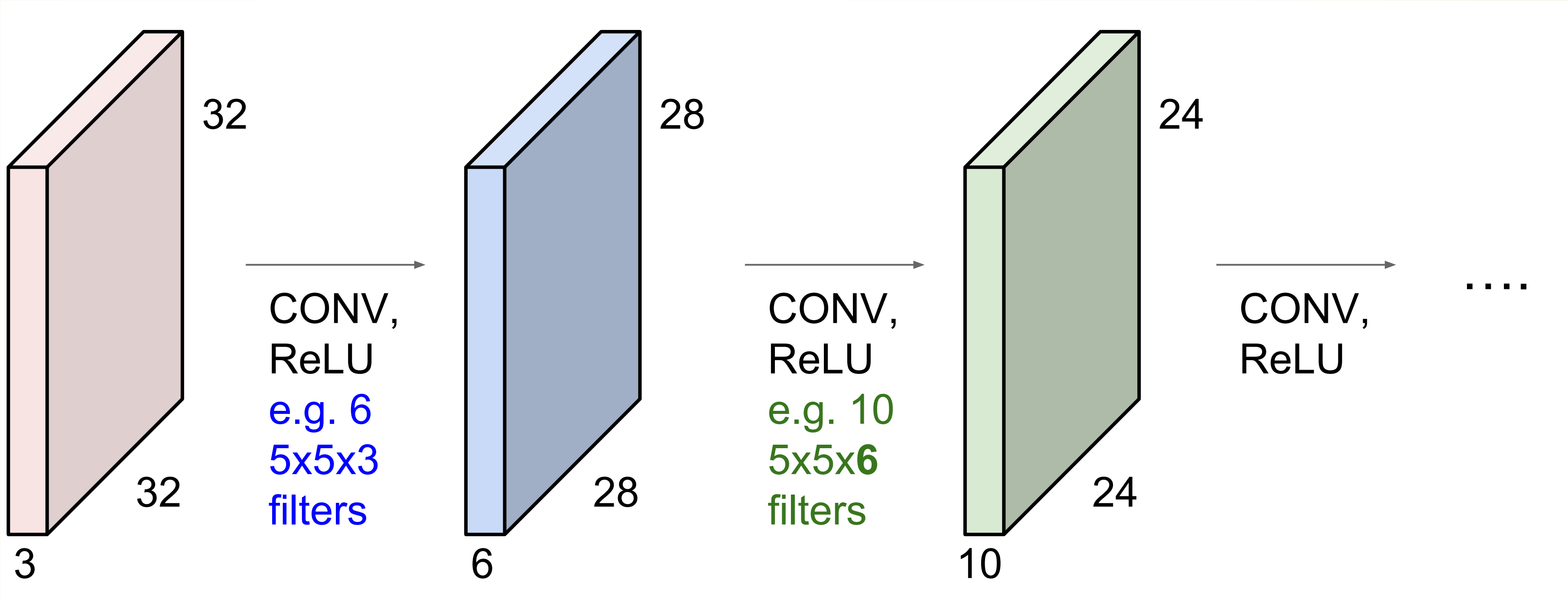
A Closer Look at Convolution: Padding



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

ConvNet:



Stride = 1
Padding = 0

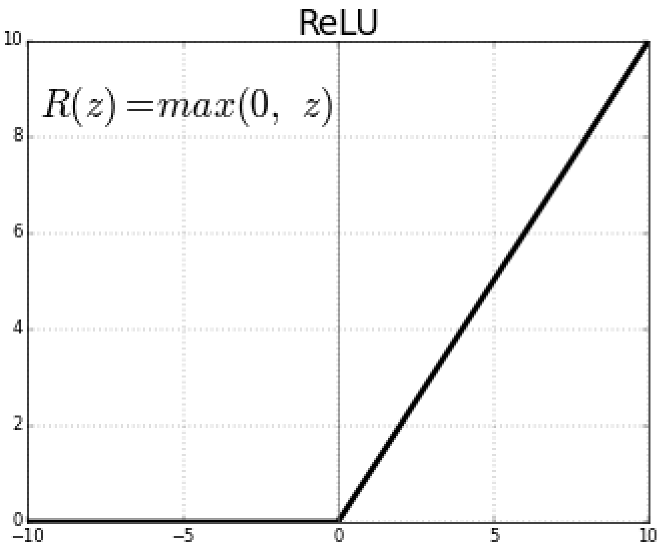
Formula: NewImageSize = floor((ImageSize – Filter + 2*Padding)/Stride + 1)

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

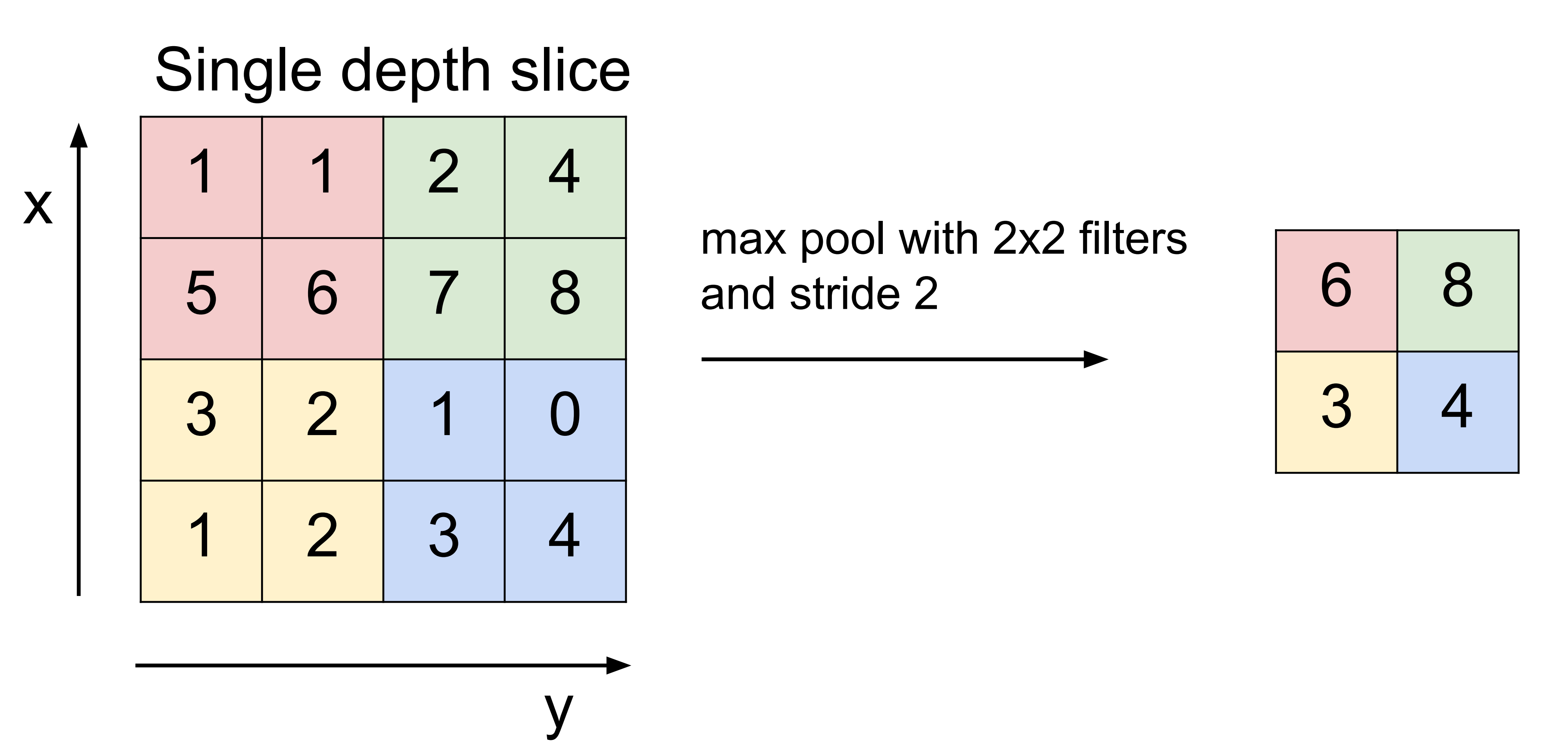
ReLU

- Non-saturating function and therefore faster convergence when compared to other nonlinearities
- Problem of dying neurons



Source: `https://ml4a.github.io/ml4a/neural_networks/`

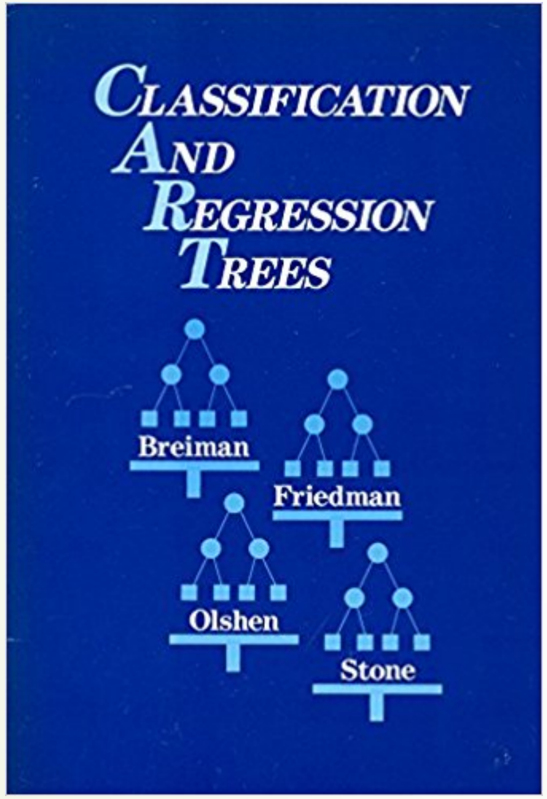
Max Pooling



Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
max pool with 2x2 filters
and stride 2
6 8
3 4

2000-2010: The Era of SVM, Boosting, … as nights of Neural Networks

Decision Trees and Boosting

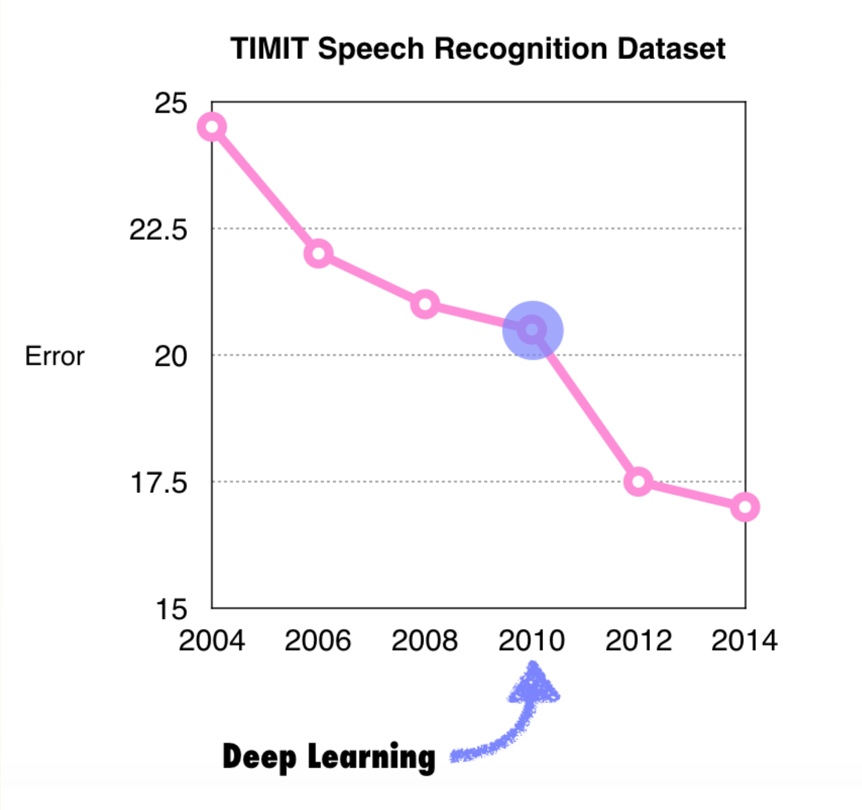


- Breiman, Friedman, Olshen, Stone, (1983): CART
- ``The Boosting problem'' (M. Kearns & L. Valiant): **Can a set of weak learners create a single strong learner?** (三个臭皮匠顶个诸葛亮？)
- Breiman (1996): Bagging
- Freund, Schapire (1997): AdaBoost
- Breiman (2001): Random Forests



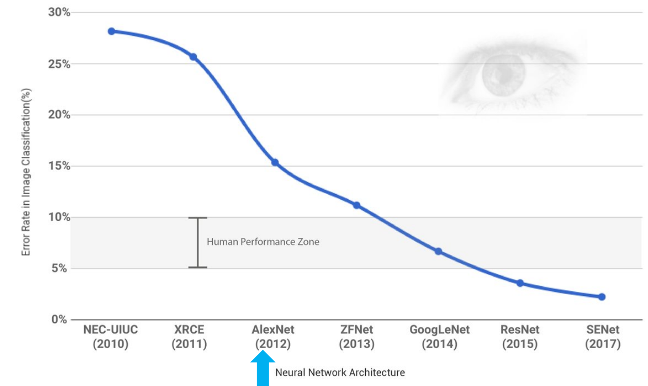
Around the year of 2012...

Speech Recognition: TIMIT



Computer Vision: ImageNet

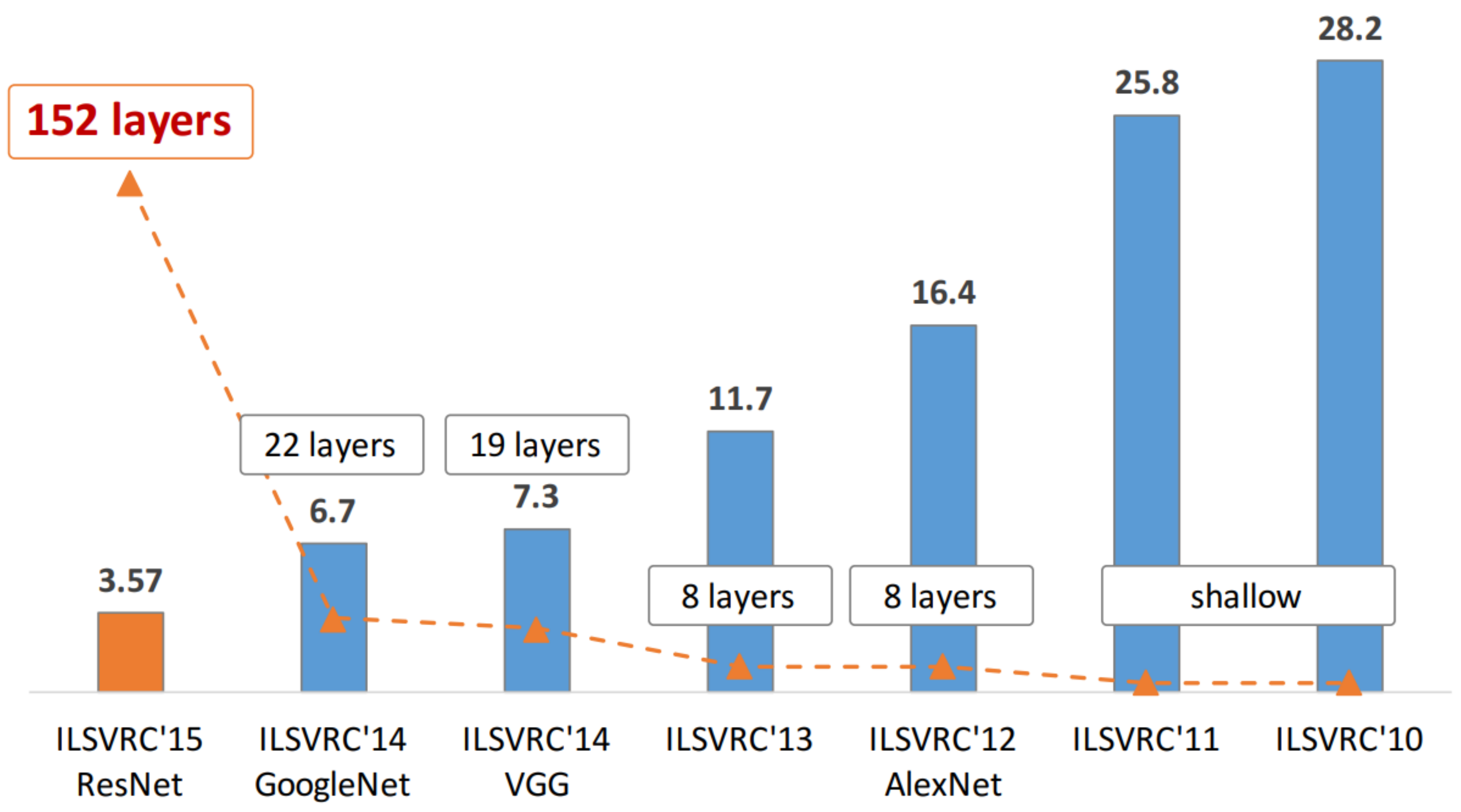
- ImageNet (subset):
  - 1.2 million training images
  - 100,000 test images
  - 1000 classes
- ImageNet large-scale visual recognition Challenge



source: https://www.linkedin.com/pulse/must-read-path-breaking-papers-image-classification-muktabh-mayank

Deep Learning

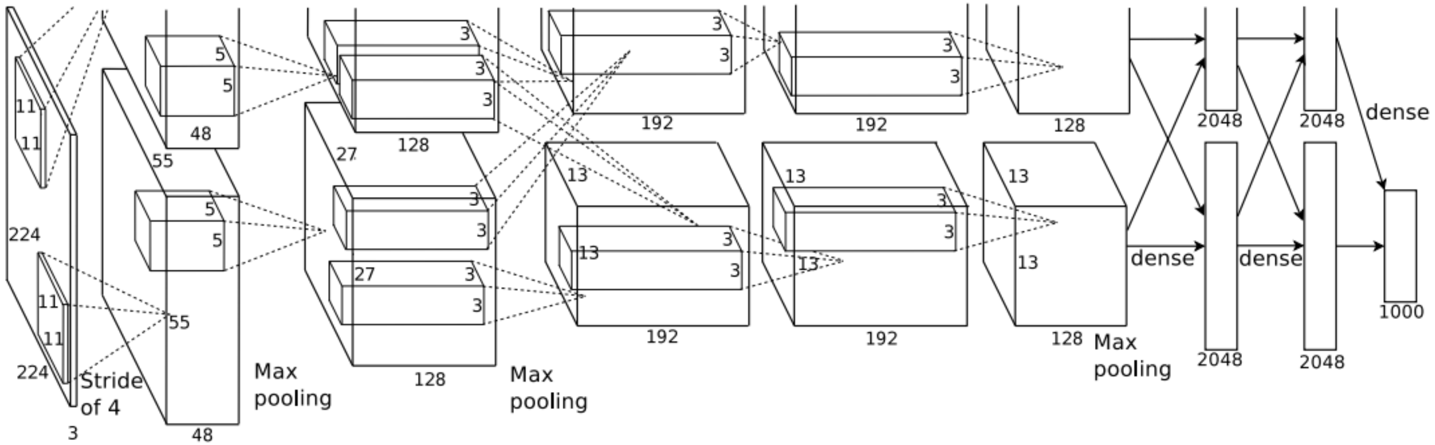
Depth as function of year



[He et al., 2016]

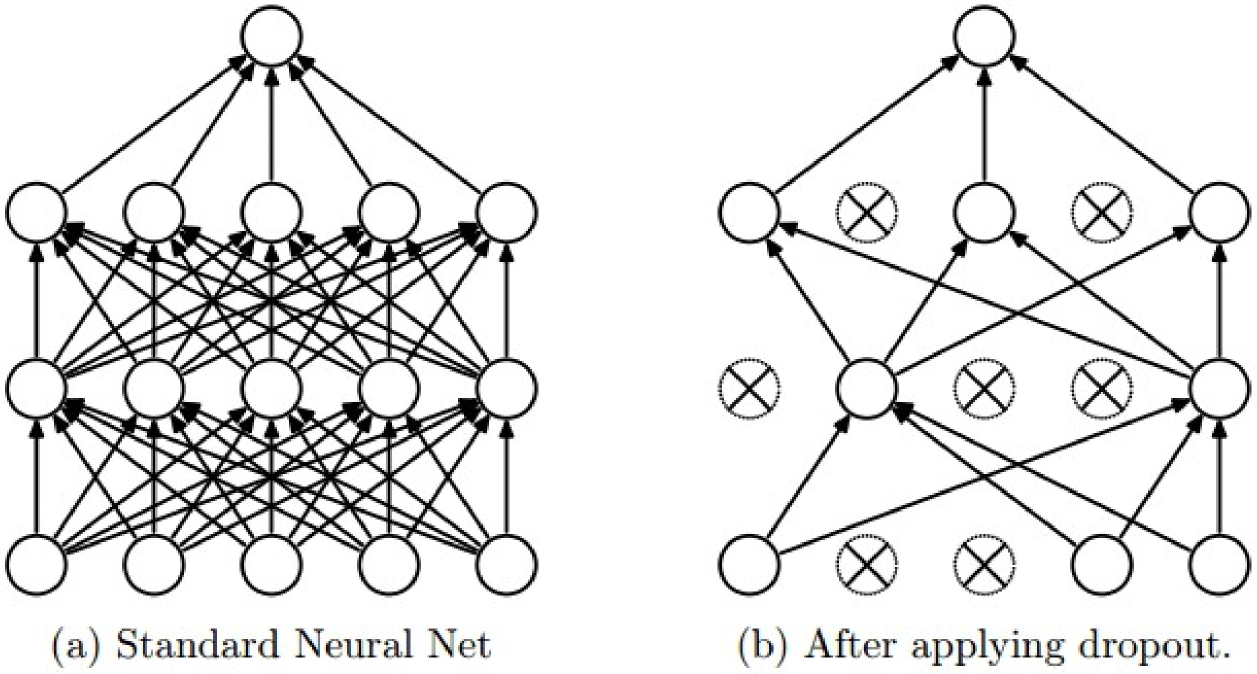
AlexNet (2012): Architecture

- 8 layers: first 5 convolutional, rest fully connected
- ReLU nonlinearity
- Local response normalization
- Max-pooling
- Dropout



Source: [Krizhevsky et al., 2012]

AlexNet (2012): Dropout

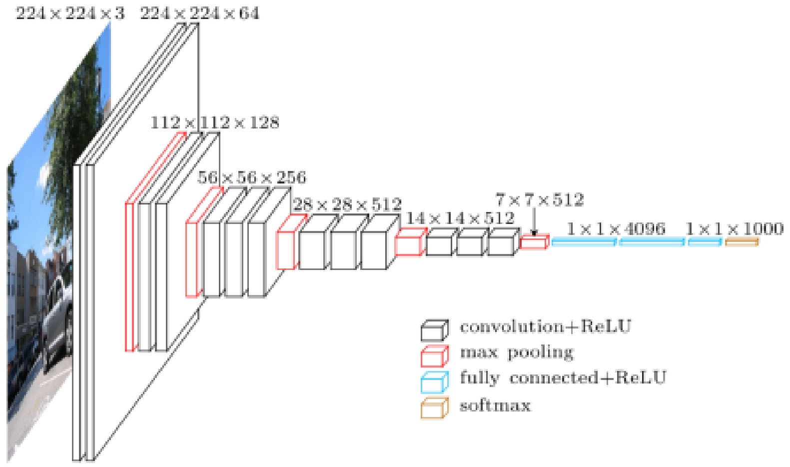


Source: [Srivastava et al., 2014]

- Zero every neuron with probability $1 - p$
- At test time, multiply every neuron by p

VGG (2014) [Simonyan-Zisserman'14]

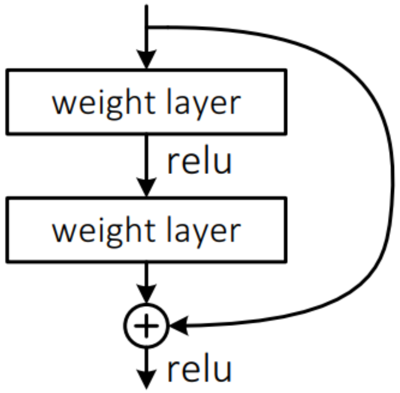
- Deeper than AlexNet: 11-19 layers versus 8
- No local response normalization
- Number of filters multiplied by two every few layers
- Spatial extent of filters 3×3 in all layers
- Instead of 7×7 filters, use three layers of 3×3 filters
  - Gain intermediate nonlinearity
  - Impose a regularization on the 7×7 filters



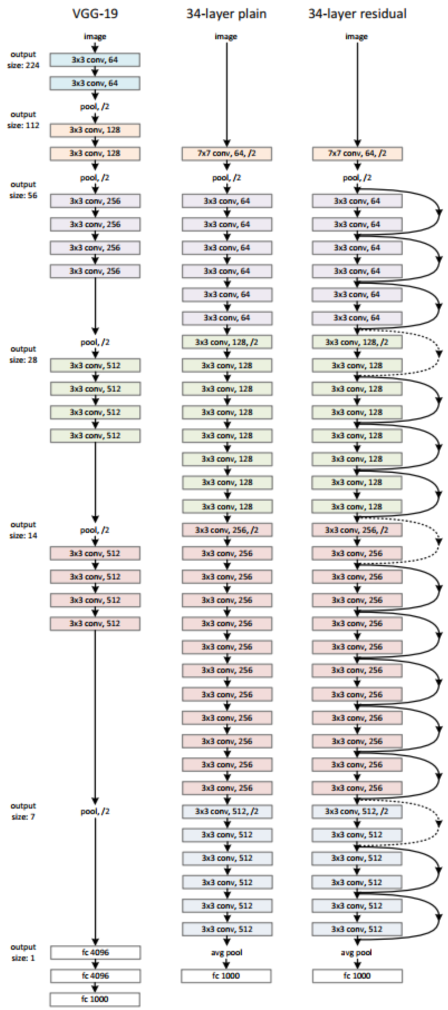
Source: `https://blog.heuritech.com/2016/02/29/`

ResNet (2015) [HGRS-15]

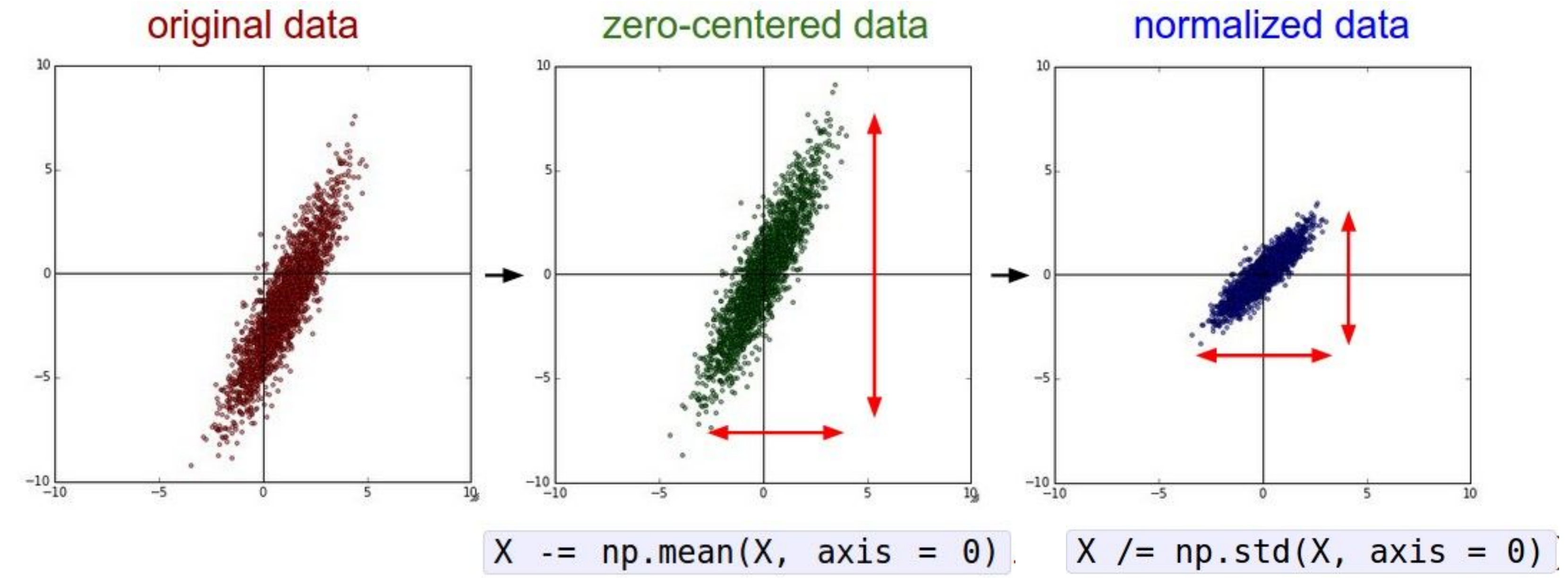
- Solves problem by adding skip connections
- Very deep: 152 layers
- No dropout
- Stride
- Batch normalization



Source: Deep Residual Learning for Image Recognition



Batch Normalization



(Assume X [NxD] is data matrix,
each example in a row)

Batch Normalization

Algorithm 2 Batch normalization [Ioffe and Szegedy, 2015]

Input: Values of x over minibatch $x_1 \dots x_B$, where x is a certain channel in a certain feature vector

Output: Normalized, scaled and shifted values $y_1 \dots y_B$

1: $\mu = \frac{1}{B} \sum_{b=1}^{B} x_b$

2: $\sigma^2 = \frac{1}{B} \sum_{b=1}^{B} (x_b - \mu)^2$

3: $\hat{x}_b = \frac{x_b - \mu}{\sqrt{\sigma^2 + \epsilon}}$

4: $y_b = \gamma \hat{x}_b + \beta$

- Accelerates training and makes initialization less sensitive
- Zero mean and unit variance feature vectors

BatchNorm at Test

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\ldots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_\mathcal{B} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_\mathcal{B}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m} (x_i - \mu_\mathcal{B})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_\mathcal{B}}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \qquad \text{// normalize}$$

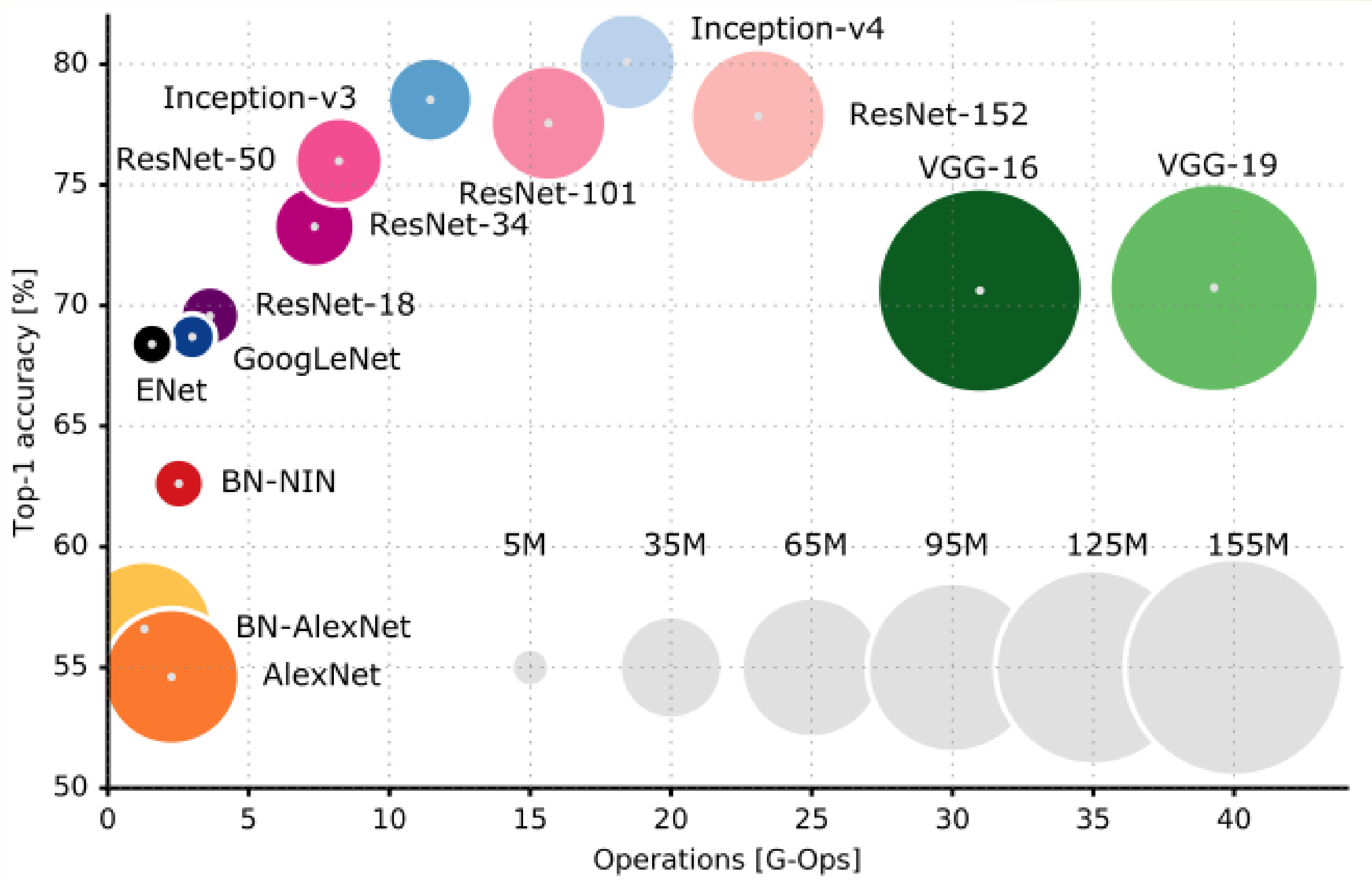
$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Note: at test time BatchNorm layer functions differently:

The mean/std are not computed based on the batch. Instead, a single fixed empirical mean of activations during training is used.

(e.g. can be estimated during training with running averages)

Complexity vs. Accuracy of Different Networks



Deep Learning Softwares

- **Pytorch** (developed by Yann LeCun and Facebook):
  - http://pytorch.org/tutorials/
- Tensorflow (developed by Google based on Caffe)
  - https://www.tensorflow.org/tutorials/
- Theano (developed by Yoshua Bengio)
  - http://deeplearning.net/software/theano/tutorial/
- **Keras (based on Tensorflow or Pytorch)**
  - https://www.manning.com/books/deep-learning-with-python?a_aid=keras&a_bid=76564dff

Show some examples by jupyter notebooks

Thank you!