

# Capstone Projec $\dagger$ for Data Science 

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## Course Infomation

- Course web:
- https://yao-lab.github.io/capstone/2020.fall/
- Time:
- TuTh, 4:30-5:50pm
- Venue:
- Zoom Meetings, join from CANVAS
- Instructor:
- Yuan Yao < yuany@ust.hk > (https://yao-lab.github.io/)
- Teaching Assistant:
- Weizhi Zhu [wzhuai@connect.ust.hk](mailto:wzhuai@connect.ust.hk)


## Course Requirement

- Prerequisite: (Statistical) Machine Learning, e.g. working knowledge about linear regression, classification, logistic regression, decision trees (CART), boosting, random forests, support vector machines, neural networks
- With these, you can pursue basic projects in this class
- Advanced project needs convolutional Neural Networks
- Projects:
- Basic level:
- Kaggle Contest: Home Credit Default Risk
- https://www.kaggle.com/c/home-credit-default-risk/overview
- Advanced level:
- Kaggle contest: Nexperia Image Classification
- https://www.kaggle.com/c/semi-conductor-image-classification-first

A Brief History of Al, Machine Learning, and Deep Learning

## Artificial Intelligence, Machine Learning, and Deep Learning

- Al is born in 1950s, when a handful of pioneers from the nascent field of computer science started asking whether computers could be made to "think"-a question whose ramifications we're still exploring today.



## Machine Learning is a new paradigm of computer programming

- During 1950s-1980s, two competitive ideas of realizing Al exist
- Rule based inference, or called Expert System
- Statistics based inference, or called Machine Learning
- 1990s- Machine Learning becomes dominant



## The $1^{\text {st }}$ machine learning method: Least Squares

- Invention:
- Carl Friederich Gauss (~1795/1809/1810),
- Adrien-Marie Legendre (1805)
- Robert Adrain (1808)
- Application:
- Prediction of the location of asteroid Ceres after it emerged from behind the sun (Franz Xaver von Zach 1801)
- Orbits of planets, Newton Laws
- Statistics,



## The $1^{\text {st }}$ neural network: Perceptron

- Invented by Frank Rosenblatt (1957)



## The Perceptron Algorithm for classification

$$
\ell(w)=-\sum_{i \in \mathcal{M}_{w}} y_{i}\left\langle w, \mathbf{x}_{i}\right\rangle, \quad \mathcal{M}_{w}=\left\{i: y_{i}\left\langle\mathbf{x}_{i}, w\right\rangle<0, y_{i} \in\{-1,1\}\right\}
$$

The Perceptron Algorithm is a Stochastic Gradient Descent method (Robbins-Monro 1951):

$$
\begin{aligned}
w_{t+1} & =w_{t}-\eta_{t} \nabla_{i} \ell(w) \\
& =\left\{\begin{array}{lr}
w_{t}-\eta_{t} y_{i} \mathbf{x}_{i}, & \text { if } y_{i} w_{t}^{T} \mathbf{x}_{i}<0 \\
w_{t}, & \text { otherwise }
\end{array}\right.
\end{aligned}
$$

## Finiteness of Stopping Time and Margin

The perceptron convergence theorem was proved by Block (1962) and Novikoff (1962). The following version is based on that in Cristianini and Shawe-Taylor (2000).

Theorem 1 (Block, Novikoff). Let the training set $S=\left\{\left(\mathbf{x}_{1}, t_{1}\right), \ldots,\left(\mathbf{x}_{n}, t_{n}\right)\right\}$ be contained in a sphere of radius $R$ about the origin. Assume the dataset to be linearly separable, and let $\mathbf{w}_{\mathrm{opt}},\left\|\mathbf{w}_{\mathrm{opt}}\right\|=1$, define the hyperplane separating the samples, having functional margin $\gamma>0$. We initialise the normal vector as $\mathbf{w}_{0}=0$. The number of updates, $k$, of the perceptron algorithms is then bounded by

$$
\begin{equation*}
k \leq\left(\frac{2 R}{\gamma}\right)^{2} \tag{10}
\end{equation*}
$$



Input ball: $\quad R=\max \left\|\mathbf{x}_{i}\right\|$.
Margin: $\quad \gamma:=\min _{i} y_{i} f\left(x_{i}\right)$

## Hillbert's 13th Problem

Algebraic equations (under a suitable transformation) of degree up to 6 can be solved by functions of two variables. What about

$$
x^{7}+a x^{3}+b x^{2}+c x+1=0 ?
$$

Hilbert's conjecture: $x(a, b, c)$ cannot be expressed by a superposition (sums and compositions) of bivariate functions.

Question: can every continuous (analytic, $C^{\infty}$, etc) function of $n$ variables be represented as a superposition of continuous (analytic, $C^{\infty}$, etc) functions of $n-1$ variables?

Theorem (D. Hilbert)
There is an analytic function of three variables that cannot be expressed as a superposition of bivariate ones.

## Kolmogorov's Superposition Theorem

Theorem (A. Kolmogorov, 1956; V. Arnold, 1957)
Given $n \in \mathbb{Z}^{+}$, every $f_{0} \in C\left([0,1]^{n}\right)$ can be reprensented as

$$
f_{0}\left(x_{1}, x_{2}, \cdots, x_{n}\right)=\sum_{q=1}^{2 n+1} g_{q}\left(\sum_{p=1}^{n} \phi_{p q}\left(x_{p}\right)\right)
$$

where $\phi_{p q} \in C[0,1]$ are increasing functions independent of $f_{0}$ and $g_{q} \in C[0,1]$ depend on $f_{0}$.

- Can choose $g_{q}$ to be all the same $g_{q} \equiv g$ (Lorentz, 1966).
- Can choose $\phi_{p q}$ to be Hölder or Lipschitz continuous, but not $C^{1}$ (Fridman, 1967).
- Can choose $\phi_{p q}=\lambda_{p} \phi_{q}$ where $\lambda_{1}, \cdots, \lambda_{n}>0$ and $\sum_{p} \lambda_{p}=1$ (Sprecher, 1972).
If $f$ is a multivariate continuous function, then $f$ can be written as a superposition of composite functions of mixtures of continuous functions of single variables:
finite composition of continuous functions of a single variable and the addition.


## Locality or Sparsity of Computation

Minsky and Papert, 1969
Perceptron can't do XOR classification
Perceptron needs infinite global
information to compute connectivity


Locality or Sparsity is important:
Locality in time?
Locality in space?

Expanded Edition


Marvin L. Minsky
Seymour A. Papert

# Multilayer Perceptrons (MLP) and Back-Propagation (BP) Algorithms 

## D.E. Rumelhart, G. Hinton, R.J. Williams (1986)

Learning representations by back-propagating errors, Nature, 323(9): 533-536

BP algorithms as stochastic gradient descent algorithms (Robbins-Monro 1950; Kiefer-
Wolfowitz 1951) with Chain rules of Gradient maps

Deep network may classify XOR. Yet topology?


We address complexity and geometric invariant properties first.


# Parallel Distributed Processing 

## by Rumelhart and McClelland, 1986

Minsky and Papert set out to show which functions can and cannot be computed by this class of machines. They demonstrated, in particular, that such perceptrons are unable to calculate such mathematical functions as parity (whether an odd or even number of points are on in the retina) or the topological function of connectedness (whether all points that are on are connected to all other points that are on either directly or via other points that are also on) without making use of absurdly large numbers of predicates. The analysis is extremely elegant and demonstrates the importance of a mathematical approach to analyz-
of multilayer networks that compute parity). Similarly, it is not difficult to develop networks capable of solving the connectedness or inside/outside problem. Hinton and Sejnowski have analyzed a version of such a network (see Chapter 7).

Essentially, then, although Minsky and Papert were exactly correct in their analysis of the one-layer perceptron, the theorems don't apply to systems which are even a little more complex. In particular, it doesn't apply to multilayer systems nor to systems that allow feedback loops.

## BP algorithm = Gradient Descent Method

- Training examples $\left\{x_{0}^{i}\right\}_{i=1}^{n}$ and labels $\left\{y^{i}\right\}_{i=1}^{n}$
- Output of the network $\left\{x_{L}^{i}\right\}_{i=1}^{m}$
- Objective Square loss, cross-entropy loss, etc.

$$
\begin{equation*}
J\left(\left\{W_{l}\right\},\left\{b_{l}\right\}\right)=\frac{1}{n} \sum_{i=1}^{n} \frac{1}{2}\left\|y^{i}-x_{L}^{i}\right\|_{2}^{2} \tag{1}
\end{equation*}
$$

- Gradient descent

$$
\begin{aligned}
W_{l} & =W_{l}-\eta \frac{\partial J}{\partial W_{l}} \\
b_{l} & =b_{l}-\eta \frac{\partial J}{\partial b_{l}}
\end{aligned}
$$

In practice: use Stochastic Gradient Descent (SGD)

## Derivation of BP: Lagrangian Multiplier

 LeCun et al. 1988Given $n$ training examples $\left(I_{i}, y_{i}\right) \equiv$ (input,target) and $L$ layers

- Constrained optimization

$$
\begin{array}{ll}
\min _{W, x} & \sum_{i=1}^{n}\left\|x_{i}(L)-y_{i}\right\|_{2} \\
\text { subject to } & x_{i}(\ell)=f_{\ell}\left[W_{\ell} x_{i}(\ell-1)\right], \\
& i=1, \ldots, n, \quad \ell=1, \ldots, L, x_{i}(0)=I_{i}
\end{array}
$$

- Lagrangian formulation (Unconstrained)

$$
\begin{aligned}
& \min _{W, x, B} \mathcal{L}(W, x, B) \\
\mathcal{L}(W, x, B)=\sum_{i=1}^{n} \quad & \left\{\left\|x_{i}(L)-y_{i}\right\|_{2}^{2}+\right. \\
& \left.\sum_{\ell=1}^{L} B_{i}(\ell)^{T}\left(x_{i}(\ell)-f_{\ell}\left[W_{\ell} x_{i}(\ell-1)\right]\right)\right\}
\end{aligned}
$$

## BP Algorithm: Forward Pass

- Cascade of repeated [linear operation followed by coordinatewise nonlinearity]'s
- Nonlinearities: sigmoid, hyperbolic tangent, (recently) ReLU.

```
Algorithm 1 Forward pass
Input: \(x_{0}\)
Output: \(x_{L}\)
    1: for \(\ell=1\) to \(L\) do
2: \(\quad x_{\ell}=f_{\ell}\left(W_{\ell} x_{\ell-1}+b_{\ell}\right)\)
3: end for
```


## back-propagation - derivation

- $\frac{\partial \mathcal{L}}{\partial B}$


## Forward pass

$$
x_{i}(\ell)=f_{\ell}[\underbrace{W_{\ell} x_{i}(\ell-1)}_{A_{i}(\ell)}] \quad \ell=1, \ldots, L, \quad i=1, \ldots, n
$$

$$
\text { - } \frac{\partial \mathcal{L}}{\partial x}, z_{\ell}=\left[\nabla f_{\ell}\right] B(\ell)
$$

Backward (adjoint) pass

$$
\begin{aligned}
& z(L)=2 \nabla f_{L}\left[A_{i}(L)\right]\left(y_{i}-x_{i}(L)\right) \\
& z_{i}(\ell)=\nabla f_{\ell}\left[A_{i}(\ell)\right] W_{\ell+1}^{T} z_{i}(\ell+1) \quad \ell=0, \ldots, L-1
\end{aligned}
$$

- $W \leftarrow W+\lambda \frac{\partial \mathcal{L}}{\partial W}$


## Weight update

$W_{\ell} \leftarrow W_{\ell}+\lambda \sum_{i=1}^{n} z_{i}(\ell) x_{i}^{T}(\ell-1)$

## Convolutional Neural Networks: shift invariances and locality



- Can be traced to Neocognitron of Kunihiko Fukushima (1979)
- Yann LeCun combined convolutional neural networks with back propagation (1989)
- Imposes shift invariance and locality on the weights
- Forward pass remains similar
- Backpropagation slightly changes - need to sum over the gradients from all spatial positions



## Max-Margin Classifier (SVM)

$$
\operatorname{minimize}_{\beta_{0}, \beta_{1}, \ldots, \beta_{p}}\|\beta\|^{2}:=\sum_{j} \beta_{j}^{2}
$$

subject to $y_{i}\left(\beta_{0}+\beta_{1} x_{i 1}+\ldots+\beta_{p} x_{i p}\right) \geq 1$ for all $i$


## Separable two classes with Max-Margin Solution



Vladmir Vapnik, 1994

## MNIST Dataset Test Error LeCun et al. 1998



Dark era for NN: 1998-2012


## 2000-2010: The Era of SVM, Boosting, ...

 as niahts of Neural Networks

## Decision Trees and Boosting


－Breiman，Friedman，Olshen，Stone，（1983）：CART
－＂＇The Boosting problem＂（M．Kearns \＆L．Valiant）： Can a set of weak learners create a single strong learner？（三个臭皮匠顶个诸葛亮？）
－Breiman（1996）：Bagging
－Freund，Schapire（1997）：AdaBoost（＂the best off－ the－shelf algorithm＂by Breiman）
－Breiman（2001）：Random Forests

## Around the year of 2012: return of NN as 'deep learning'

Speech Recognition: TIMIT


Deep Learning

Computer Vision: ImageNe†

ImageNet


## Depth as function of year



## Reaching Human Performance Level in Games

May 11th, 1997
Computer won world champion of chess
(Garry Kasparov)


Deep Blue in 1997


AlphaGo "LEE" 2016



## Number of Al papers on arXiv, 20102019

Number of Al papers on arXiv, 2010-2019
Source: arXiv, 2019


## Growth of Deep Learning

'Deep Learning' is coined by Hinton et al. in their Restricted Boltzman Machine paper, Science 2006, not yet popular until championing ImageNet competitions.


## Some Cold Water: Tesla Autopilot Misclassifies Truck as Billboard



Problem: Why? How can you trust a blackbox?

## Deep Learning may be fragile in generalization against noise!


$+.007 \times$

$\operatorname{sign}\left(\nabla_{\boldsymbol{x}} J(\boldsymbol{\theta}, \boldsymbol{x}, y)\right)$
"nematode" $8.2 \%$ confidence

$\boldsymbol{x}+$ $\epsilon \operatorname{sign}\left(\nabla_{\boldsymbol{x}} J(\boldsymbol{\theta}, \boldsymbol{x}, y)\right)$
"gibbon" $99.3 \%$ confidence
[Goodfellow et al., 2014]

"black hole"

## CNN learns texture features, not shapes


(a) Texture image
81.4\%
10.3\%
8.2\%

Indian elephant
indri
black swan

(b) Content image
71.1\% tabby cat
17.3\% grey fox
3.3\% Siamese cat

(c) Texture-shape cue conflict 63.9\% Indian elephant 26.4\% indri 9.6\% black swan

## Overfitting causes privacy leakage

- Model inversion attack leaks privacy


Figure: Recovered (Left), Original (Right)
Fredrikson et al. Proc. CCS, 2016

## What's wrong with deep learning?

Ali Rahimi NIPS'17: Machine (deep) Learning has become alchemy. https://www.youtube.com/watch?v=ORHFOnaEzPc

Yann LeCun CVPR'15, invited talk: What's wrong with deep learning? One important piece: missing some theory (clarity in understanding)!
http\%/Itechtalks.tv/talks/whats-wrong-with-deep-learning/61639/


Being alchemy is certainly not a shame, not wanting to work on
advancing to chemistry is a shame! -- by Eric Xing

## Shall we see soon an emergence from Alchemy to Science in deep leaning?

How can we teach our students in the next generation science rather than alchemy?

## In this class

- Understand its principles: statistics, optimization
- Analyze the real world data with the methods
- Team-work (no more than 3 persons per team)!

Thank you!


