# Dimensionality Reduction of Face Order Problem Using Non-linear Embedding Methods

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## Introduction

The exploratory data analysis and visualization are of great importance in many areas of science and engineering. While in these areas, large amounts of data requires to be analyzed. Therefore, a deep understanding of the dimensionality reduction is needed in practical applications. The images of a person's face observed under different pose and lighting conditions can be considered as points in a high-dimensional vector space. In order to judge the similarities and detect the differences of these images, the dimensionality reduction is required.

In this project, we order the faces by using the four algothrims, i.e., Diffusion map, MDS-embedding , ISOMAP-embedding and LLE-embedding. It is found that all the four algothrims can effectively capture the Euclidean structure of the dataset. Besides, the ISOMAP is also capable of discovering the nonlinear degrees of freedom of the dataset.

## Methods and Materials

●**Data description:** The dataset contains 33 faces of the same person (Y∈R112×92×33) in different angles. The pictures of the 33 faces are shown below. To do data analysis, a data matrix $X \in \mathrm{R}^{n \times p}$ , where n=33 and p=10304 is created.

●**Diffusion map:** Diffusion map is a non-linear dimensionality reduction technique. The main idea is to map coordinates between data and diffusion space to reorganize data according to the diffusion metric. In the diffusion space the Euclidean distance approximates the diffusion distance thus the dataset's intrinsic underlying geometry can be preserved while reducing dimensionality.

●**MDS embedding:** MDS is used to translate information about the pairwise distance among a set of *n* individuals into a configuration of *n* points mapped into the Cartesian space. This algorithm places each object into N-dimensional space. The goal of MDS is to find *I* vectors $\in \mathrm{R}^N$ and then find an embedding from the *I* objects into $\mathrm{R}^N$. In the classical MDS algothrim, first, $B = -H \cdot D \cdot H^T / 2$ , where *H* is a centering matrix is computed. Second, Eigenvalue decomposition is computed. Third, choose top *k* nonzero eigenvalues and corresponding eigenvector $\tilde{X}_k = U_k \Lambda_k^{1/2}$ , where $U_k = [u_1, \ldots\ldots, u_k], \quad u_k \in R^n$ and $\Lambda_k = \mathrm{dig}(\lambda_1, \ldots\ldots, \lambda_k)$ with $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_k > 0$ are computed.

Fig.1. 33 faces of the same person in the dataset

## Methods and Materials

● **ISOMAP-embedding**: ISOMAP is an extension of MDS, where pairwise Euclidean distance data points are replaced by geodesic distances, computed by graph shortest path distances. In this algorithm, first, the graph shortest path distances $d_{ij} = \min_{P=(x_i \ldots x_j)} \left( \|x_i - x_{t1}\| + \ldots + \|x_{tk-1} - x_j\| \right)$ is computed. Second, $K = -H \cdot D \cdot H^T (D : [d_{ij}^2])$ is computed. Third, the eigenvalue decomposition is computed. Last, the top *d* nonzero eigenvalues and corresponding eigenvectors $Y_d = U_d \Lambda_d^{1/2}$ are computed.

●**LLE-embedding**: This algorithm assumes that any data point in a high dimensional ambient space can be a linear combination of data points in its neighborhood. It is a local method as it involves data points in local neighbors and hence a sparse eigenvector decomposition.

## Results

### Diffusion map

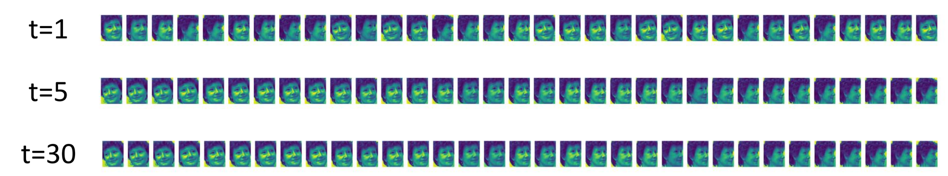


Fig.2(a). Ordered faces with different time scale along first diffusion coordinate

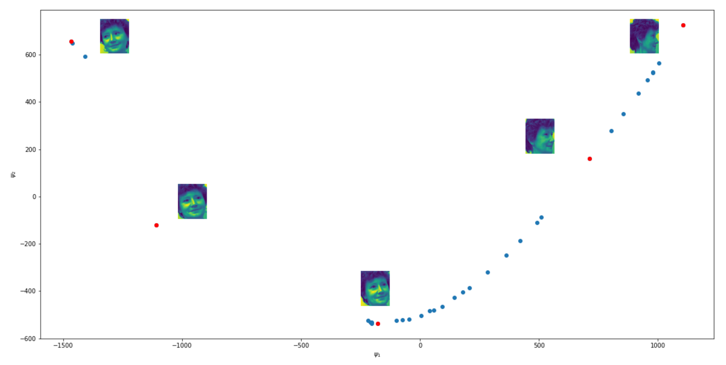

Fig.2(b). Diffusion map embedding(t=30)

In diffusion map, time scale parameter t highly influences the results of this method. Large t can better explore the intrinsic similarities among different images but require more computational cost. As is shown in the left figure, for t=1 the faces are not reasonably ordered based on the angles. When t increases to 5, they are much better ordered.

Along the second diffusion coordinate, no clear meaning of the ordered faces can be found yet.

### MDS embedding

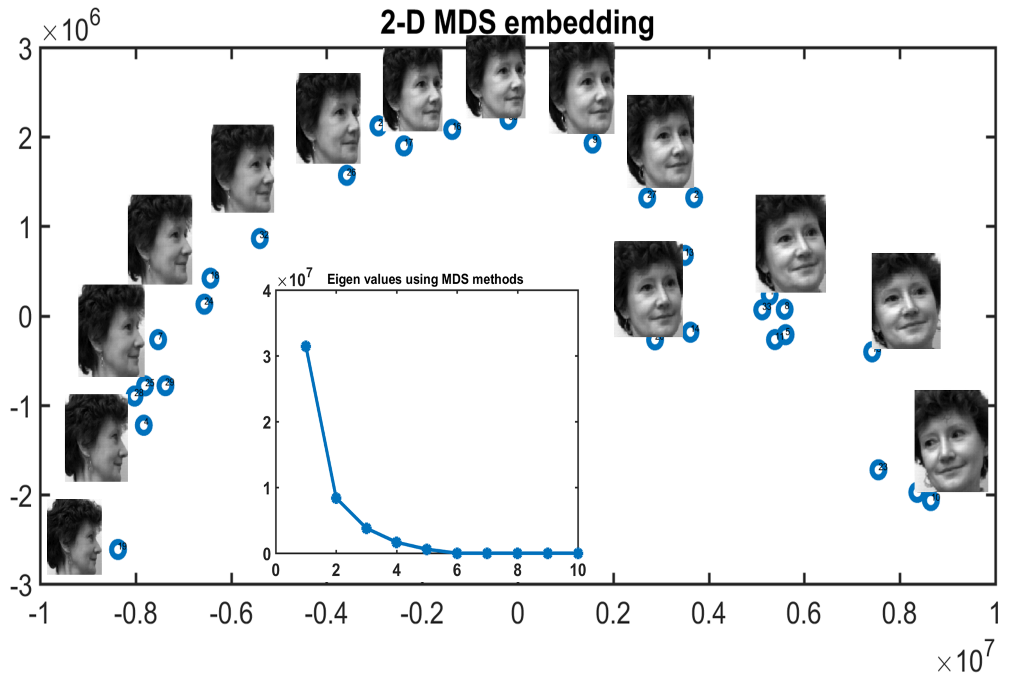


Fig.3. MDS embedding (inset is eigenvalue plot)

In MDS, the first two components are used because the first two components explain roughly 80% of the total variance, as indicated by the inset. It is seen that MDS can effectively capture the Euclidean structure of the faces. However, it fails to detect the intrinsic three dimensionality of the face dataset.

### Isomap

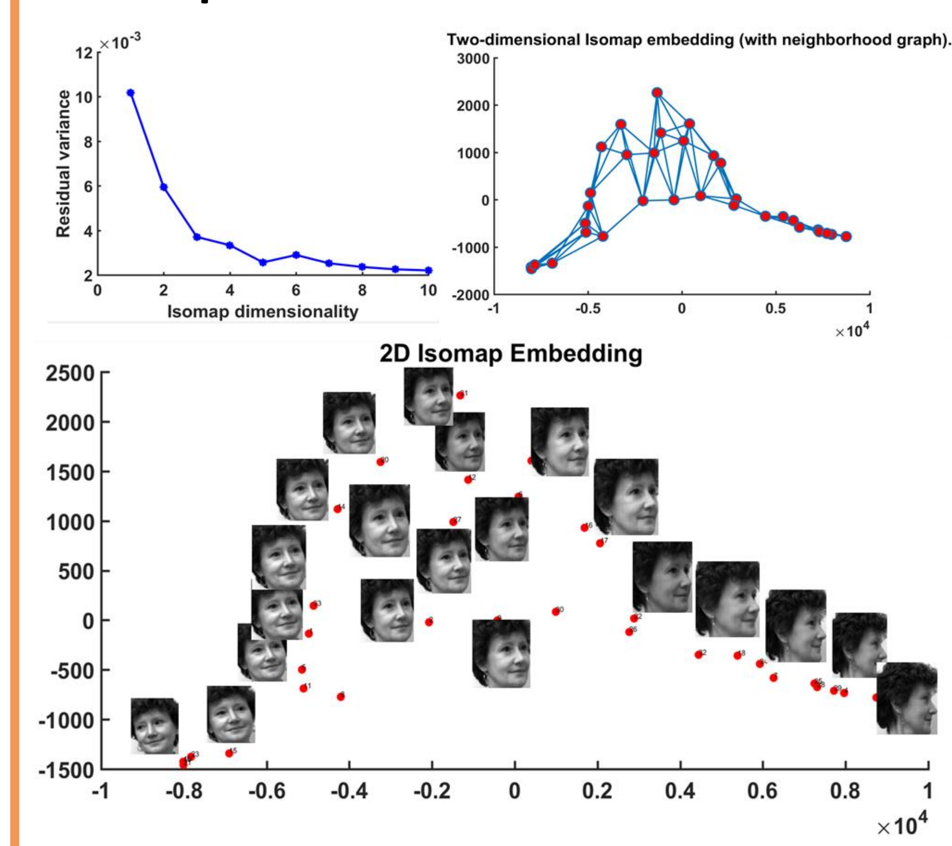


Fig.4. 2-D Isomap embedding

It is seen from Fig. 4 that the Isomap embedding can correctly detects the three degrees of freedom of the face images, while the MDS embedding shown in the previous Fig. 3 can not. In the other words, Isomap has captured the data's perceptually relevant structure.

Therefore, the Isomap algorithm can manipulate high dimensional observations in terms of their intrinsic nonlinear degrees of freedom.

To be specific, the residual variance decreased with the increase of the dimensionality, and the selected 5 neighbor can be detected in Fig.4. The selected faced of this embedding is also shown in Fig.4.

### LLE

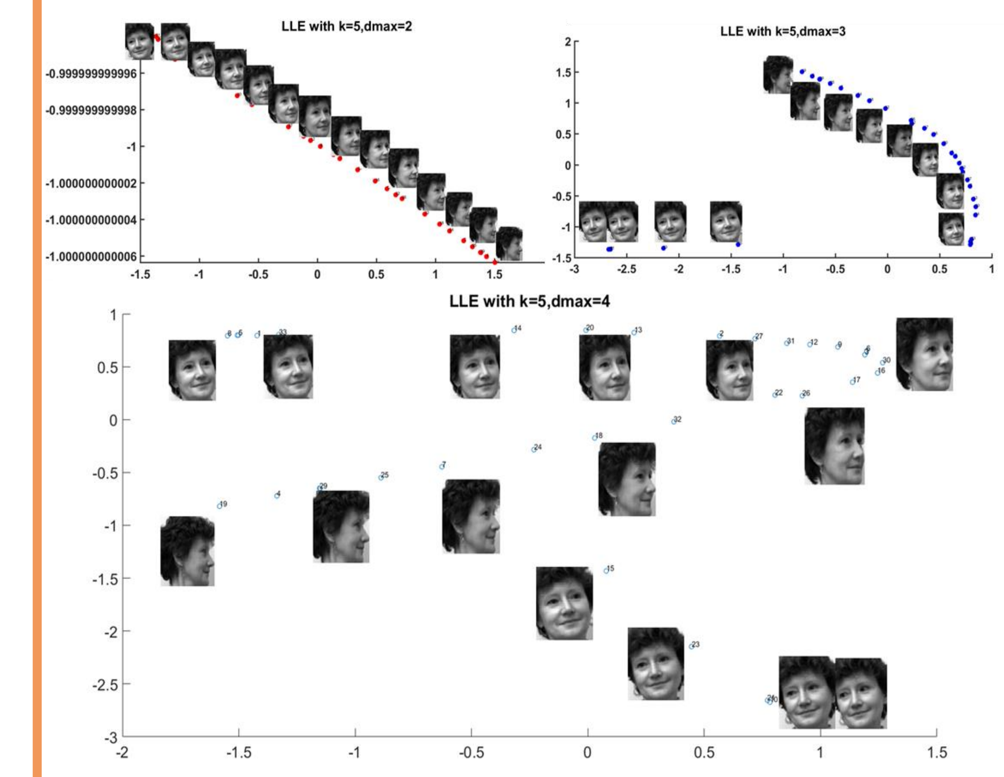


Fig.5. 2-D LLE embedding

The LLE algorithm embedded higher-dimensional data to lower dimensions by reconstructing the data from the neighboring data. In this specific face order problem, we selected 5 neighbors. We get the embedded results by varying the max embedding dimensionality(dmax). For example, dmax chosen as 2,3,4 embedding dimensionalities are plotted in Fig .5. When dmax=2, the embedding the shows a linear trend and the faces are ordered mainly based on the angles. And the plot with dmax=3 shows a similar trend. When dmax=4, the variation between these faces are maximized and the similar pictures clustered.

## Discussions &Conclusions

In this dimension reduction problem of ordering the faces, different algorithms including linear embedding (MDS) and nonlinear embedding methods(Diffsion map, isomap, LLE) are explored. As an extension of the MDS method, the Isomap captures the geometric distances and can well describe the high dimensional problem, which is superior to the MDS methods which only capture two intrinsic features. The LLE method uses the neighbouring data to reconstruct low dimensional embedding and such methods can offer information of global geometry. Diffusion map as another nonlinear method, maps the data to a diffusion space to preserve their diffusion distance. It can well distinguish different images with a relative large time scale parameter. In this specific face order problem, it seems that the non-linear embedding algorithms can better describe intrinsic structures of the data.

**Youtube link: https://youtu.be/0B3ywTWL40g**

## Acknowlegements

YU Zhijie conducted the diffusion map analysis. SUN Jing and LUO Shuang conducted Isomap, MDS, and LLE algorithm and analyzed the data. All the group members designed poster and contributed to the contents.

## References

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